

Effects of Writing-for-Learn Strategies and Triangulated Evaluation on Students' Mathematics Achievement, Academic Self-Perception, and Learning- Engagement in South-South Nigerian Teacher Education Colleges

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DECLARATION

This dissertation is the original work approved and supervised by the supervisory committee and carried out by me Mary Patrick Uko, MOUAU/PG/Ph.D/MAE/16/9472 in partial fulfilment of the requirements for the award of the Doctor of Philosophy degree in Measurement and Evaluation in the Department of Science Education.

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CERTIFICATION

We certify that this dissertation titled: Effects of writing-for-learn strategies and triangulated evaluation on students' mathematics achievement, academic self-perception, and learning- engagement in south-south Nigerian teacher education colleges, written by: Mary Patrick Uko has been examined and found acceptable for the award of Doctor of Philosophy degree in Measurement and Evaluation in the Department of Science Education.

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DEDICATION

This work is dedicated to God Almighty for giving me the grace to complete the programme. It is also dedicated to the memory of my Late Parents, Elder and Deaconess Martin Hanson Ekanem, and my Late Brother, Rev. Dr. Emmanuel Martins who started it all before their transitions.

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ABSTRACT

This study investigated the effects of writing-for-learn strategies and triangulated evaluation on students' mathematics achievement, academic self-perception, and learning- engagement in south-south Nigerian teacher education colleges, key affective outcomes in mathematics learning. A quasi-experimental design with a non-randomized control group was employed. The research was structured around nine research questions and corresponding null hypotheses. The study was conducted in the South-South geopolitical zone of Nigeria, focusing on a population of 10,648 second-year students across thirteen Colleges of Education during the 2018/2019 academic session. From this population, 3,306 students at Akwa Ibom State College of Education were considered, with a purposive and simple random sample of 386 second-year students offering Basic General Mathematics IV were selected for the study. These students were assigned to either experimental or control groups. The experimental group was taught and assessed using writing-for-learning strategies and triangulated assessment tools, while the control group experienced conventional closed-ended testing methods. Four researcher-developed instruments were used: a 25-item Mathematics Achievement Test, 40-item self-concept and interest inventories, a mathography, and an alternative solution worksheet. Both quantitative and qualitative data were collected and analyzed. Descriptive statistics (mean and standard deviation) were used to address the research questions, while inferential statistics, Multivariate Analysis of Covariance (MANCOVA), Multivariate Analysis of Variance (MANOVA), and Multiple Regression, tested the hypotheses at a 0.05 level of significance. Template analysis was employed for the qualitative data. Findings indicated that writing-to-learn and triangulated assessment approaches significantly enhanced students' mathematics achievement and affective outcomes. Female students demonstrated greater gains than males in both academic and affective domains. However, the interaction effect between writing-to-learn and triangulation approaches was not statistically significant. These findings carry meaningful implications for curriculum developers, teacher educators, mathematics lecturers, and students. It is recommended that mathematics educators incorporate writing-based strategies and performance-oriented assessments such as alternative solution worksheets, knowledge-demonstration writing tasks, and project-based methods throughout mathematics instruction. Such approaches may help bridge the gap between mathematical concepts and real-life applications, enhance retention, and change negative perceptions of mathematics. Moreover, pre-service mathematics teachers should receive training in these alternative assessment methods, and curriculum planners should embed them within revised mathematics curricula in Nigeria.

CHAPTER ONE INTRODUCTION

➤ *Background to the Study*

In academic settings, assessment represents a multifaceted process. It is a vital component in the process of teaching and learning. An assessment that is not properly done, or do not meet the requirements and the expectations of the teachers and learners, may impede the process of learning. Researchers and educators have been trying optimal ways to measure student's knowledge, competences, and performances in testing to make assessment beneficial for the learners. Lecturers use various strategies to evaluate students' progress. An assessment system that measures students performance at different classes, or class level, based on public adopted standard of what is to be taught is referred to as standard - based assessment. This assessment is designed to hold schools publicly accountable for enabling each student meeting these high standards. Standards are the centre piece of sustainable development goal (SDG), therefore a method of assessment should be employed to set this standard that will cut across the different class levels, for proper accountability and measures of growth of individual or group of students over time (Udofia and Uko, 2016). Modern educational assessment programmes are used for a variety of purposes: to Improve student learning of content standards through improved instruction based on the assessment results; to complement curriculum or teaching methods; to inform lecturers/students of their progress; to inform the public about school performance; to be used as a guide in decision making about students, lecturers, or schools; and to provide various data comparisons (Udofia and Uko, 2016). The information that serves these purposes is derived from individual tests that make up an assessment programme. The purpose of any particular assessment, however, is more specific, that is, its purpose is to give users an accurate description of what students know and are able to do. The most important characteristics of any assessment procedure is its effect on validity.

Accountability is a fundamental principle in all business-driven systems, and education is no exception. As a strategic investment, education demands accountability to ensure quality enhancement in academic standards. Educational administrators across all institutional levels are responsible for justifying the utilization of resources by demonstrating tangible outcomes to stakeholders, including the government, parents, and the broader society. Effective accountability hinges on sound assessment practices, which entail both objective measurement and value judgment. With educational accountability becoming central to instructional reforms, assessment now plays a pivotal role in evaluating student learning. Boud and Falchikov (2006) contend that traditional higher education focuses more on knowledge acquisition than on active learner participation. Boud (2000) further cautions that some prevalent assessment methods fail to adequately prepare students for continuous learning beyond the classroom.

Ogunleye and Omolayo (2016) describe assessment as the systematic collection and interpretation of data to inform decision-making. Ezenwa-Nebife (2014) emphasizes its role in tracking students' academic development, while Emaikwu (2014) frames it as a process of evaluating behavior to guide decisions related to students, curriculum, and instruction. Drawing from these perspectives, the researcher views assessment as a comprehensive approach that involves gathering, describing, and analyzing performance-related information. Integrating multiple assessment forms into a unified framework generates robust and reliable insights that guide decisions concerning students, teachers, institutions, and policymakers. Crucially, assessment can also serve as a tool to enhance instruction, promote learning, and support lifelong educational development. In many mathematics classrooms, pedagogical strategies emphasize rote memorization and procedural repetition rather than fostering analytical and conceptual thinking. Students are rarely encouraged to articulate their thought processes or engage collaboratively to deepen their mathematical reasoning (Inekwe, 2019). Female students, in particular, have historically underperformed in mathematics compared to their male counterparts. Ker (2013) reports that quality mathematics education is critical in nurturing students' interests, values, and skills required for pursuing STEM-related careers.

The National Council of Teachers of Mathematics (NCTM, 2000; 2013; 2014) advocates for the use of diverse and alternative assessment methods that provide all students with meaningful opportunities to learn and apply mathematical concepts. However, many students still equate learning mathematics with memorizing formulas and solving algorithms, which limits their understanding of the subject's creative and practical aspects. To address these challenges, the NCTM (2014) emphasizes instructional principles that promote meaningful engagement. Learners should work on challenging tasks that connect prior knowledge to new concepts, confront misconceptions, build both conceptual and procedural knowledge, and receive timely, descriptive feedback to reflect on and refine their understanding. Developing metacognitive skills awareness of one's own thinking further strengthens the learning process. In Nigeria's Colleges of Education, traditional assessment primarily closed-ended, paper-based tests remains the dominant evaluation method. These tests often assess memory recall and organization under exam conditions, without prior knowledge of the questions or access to relevant materials. This overreliance on conventional testing may partly explain the prevalence of academic dishonesty among students. In response, educational reformers such as Creswell (2008), Evans and Swan (2014), and Swan and Burkhardt (2014) have called for the adoption of alternative assessment methods to better align with 21st-century learning goals.

Alternative assessments including writing-to-learn strategies, like write to demonstrate knowledge, and the performance-based assessment tools and triangulation assessment methods such as Reciprocity with peers, that is peer-based discussions (e.g., think-pair-share), and project-based evaluations or open- time methods, which will subsequently be referred to in this study as alternative assessments methods are available for utilization and have shown promise in deepening students' understanding of mathematical processes. Writing-to-learn mathematics refers to using written expression to communicate mathematical ideas and

reasoning, including explanations, reflections, and visual representations (Quinones, 2005). This strategy enhances comprehension, supports personal engagement, and helps contextualize mathematics in real-life scenarios (Stonewater, 2002). Assignments such as essays, reports, and creative tasks fall under writing-to-demonstrate-knowledge assessments. These tasks require students to synthesize and explain concepts, making their learning visible. Performance-based assessments extend this further by requiring students to apply their knowledge through real-world tasks such as presentations or analytical reports (Nitko, 2004; Adamson & Darling-Hammond, 2010).

Writing in mathematics classrooms has been linked to improved critical thinking, as it encourages students to analyze, apply, and evaluate concepts. It is writing that uses impromptu, short, or informal writing tasks designed by the lecturer and included throughout the lesson to help students think through key concepts and ideas. This process helps clarify their understanding and build connections between old and new knowledge (Nilson, 2003; O’Kelly, 2013; Palma, 2018). Through writing, students engage in problem-solving, reflection, and metacognitive inquiry, fostering deeper learning. NCTM (2009, 2014) also emphasizes that purposeful mathematical writing promotes clear articulation of ideas and helps students reflect on their learning. Describing how a problem was solved fosters conceptual clarity and strengthens mathematical reasoning. According to Pugalee (2005), reflective writing is central to learning mathematics, as it compels students to assess their reasoning and problem-solving approaches. Furthermore, writing activities give educators unique insight into students' understanding, misconceptions, and thinking patterns. Pugalee (2001) observed that these activities provide a valuable diagnostic tool for instructors while enhancing students' metacognition. Writing helps learners process information actively, enabling them to link, explain, interpret, and plan their responses.

In summary, integrating writing-to-learn strategies into mathematics education offers multiple benefits. It enhances understanding, supports metacognitive growth, fosters problem-solving skills, and enables educators to better support diverse learners. Such practices are crucial in developing learners' ability to think, reason, and communicate mathematically, skills essential for academic success and lifelong learning. Through writing activities, educators gain valuable insight into students' strengths, areas of difficulty, and interests. These activities also highlight the connections students make between writing, problem-solving, and metacognitive processes while engaging with mathematical problems (Pugalee, 2001). Writing enables students to practice critical academic skills such as inference, communication, organization, interpretation, and reflection. Within mathematics classrooms, writing serves as a vital pedagogical tool for deepening student understanding of key concepts. According to the National Council of Teachers of Mathematics (NCTM, 2009), engaging in mathematical writing requires learners to clearly articulate their thoughts and reflect meaningfully on what they have learned. Purposeful writing fosters deeper comprehension by encouraging students to analyze their thinking and refine their ideas. Scholars such as Pugalee (2004, 2005) emphasize that writing is a powerful medium through which students can learn and internalize new concepts.

This study examines the cognitive and affective dimensions of learning mathematics, with a focus on problem-solving skill development, enhanced conceptual understanding, and the impact of writing on emotional and motivational outcomes. These characteristics and processes of problem solving, affective outcomes that will be discussed and analyzed have been described and includes development of problem-solving skills, increasing conceptual understanding, demonstration of procedural application, demonstration of Mathematics reasoning, development of content connections, self-concept and interest. Self- concept has been recognized as a key Psychological factor of the learners which contributes significantly in the academic achievements. Every individual is striving either by thought or action or both to be successful, happier and better in future. It has been repeatedly established that the performance of students in academics is effectively regulated by their Self-Concept. Self-concept is a powerful internal cognition which propels one to act and sustain the action (Sarouphim and Chartouny, 2017). Failure and success also impact on self-concept. It is a key in the progress of one's life (Aida- Mehrad, 2016).

In terms of interest, motivational psychologists, developmental psychologists, and educational psychologists have provided at least three perspectives on interest: personal interest as an individual disposition, interestingness as an aspect of the context, and interest as a psychological state (Wei, 2014; Schunk, Pintrich, and Meece, 2008). The situational interest may be triggered and maintained by interestingness as a contextual factor (Renninger and Hidi, 2002; Schunk et al., 2008). To summarize the three perspectives on interest in this view, situational interest is triggered by interestingness of the academic task, and may further evolve into personal interest depending on opportunities and support available to the student and this process may provide valuable implications for effective learning and instruction (Renninger and Hidi, 2002). Nevertheless, for the study of science base courses including Mathematics, gender disparity is a factor considered by scholars to affect students' achievement in school. Notably, research suggests that female students often acquire language skills more rapidly and proficiently than their male counterparts, with gender-related differences in verbal ability emerging at early developmental stages. This linguistic advantage may influence how female learners respond to writing-based instructional strategies in mathematics. This method aligns well with the verbal and reflective strengths often exhibited by female students. Given these potentials, this research seeks to explore how writing can be utilized not only as an instructional strategy but also as a reliable tool for assessing mathematical learning from the student's perspective.

Beyond the cognitive benefits, writing also supports active learning and critical reflection. Pioneering composition theorist Janet Emig (as cited in Reilly, 2007) described writing as a uniquely integrative process involving the coordinated activity of the

hand, eye, and brain. She argued that writing activates both hemispheres of the brain and provides learners with a multisensory, representational mode of learning. Writing externalizes information, making it accessible for review, critique, and feedback. It also promotes metacognition encouraging students to reflect on and refine their understanding as they revise their written responses (Burns, 2003). Writing-to-learn strategies, in particular, require learners to engage deeply with mathematical content. This approach fosters focused thinking and personal investment in learning, giving students the opportunity to explore multiple solution strategies and critically examine their reasoning (Palmer, 2018). By documenting their problem-solving processes, students become active participants in learning and develop higher-order thinking skills (Thompson et al., 2008). To effectively understand the interplay of cognition, metacognition, and emotion in mathematics education, students need structured support to build reflective and self-monitoring capabilities. Writing provides such a platform. As Nilson (2003) asserts, writing-to-learn empowers students to reach a more personalized and profound understanding of mathematics. This study positions alternative assessment methods, particularly writing-based strategies, as catalysts for such transformative learning.

Furthermore, triangulated assessment—an approach that incorporates multiple forms of evidence enables educators to better understand students' conceptual grasp and cognitive processes. Triangulation is defined as 'the combination of methodologies in the study of the same phenomenon' (Johnson, Onwuegbuzie and Turner, 2007). Triangulation assessment method attempts to respect the multiple beliefs, perspectives and usefulness of both qualitative and quantitative approaches, incorporating the best of both worldviews (Guba and Lincoln, 2005). Creswell (2008) advances a number of strengths of Triangulation method assessment, strengths which render the approach appropriate for use. Firstly, quantitative and qualitative data together provide a better understanding of the students problem than either type by itself; secondly, one type of assessment is not enough to answer the questions of how well the students have learned the material to be learnt; and thirdly, from a practical perspective, multiple viewpoints are needed. Another aspect of triangulation method that is appealing is that one method can develop, inform and complement the other, and thereby mitigate the limitations associated with the primary method to bring about complete, comprehensive and equitable Mathematics assessment.

Throughout this study, students will be regularly instructed by the lecturers to talk to each other in structured ways (i.e., think-pair-share, turn-and-talk), to flash back at the problem, explain their mathematical thinking, and to discuss their work or their mathematical thinking with other students, that is "think aloud". This method complemented the other methods for better results. Triangulation methods provide greater breadth and depth, which facilitate enhanced description and deeper understanding (Johnson, Onwuegbuzie and Turner, 2007). The two methods used in this study corresponds to the three domain of learning, making the assessment complete, comprehensive and equitable. Mason (2006) asserts that the fusion of quantitative and qualitative ideas can create data and arguments that can form the basis for well-founded social theory. Following the Federal Government reforms in education and the need to attain the Sustainable Development Goals (SDGs) and the critical targets of the National Economic Empowerment and Development Strategies (NEEDs), which can be summarized as value-reorientation, poverty eradication, job creation, wealth generation and using education to empower the people, it has become imperative that the existing curricula for tertiary institutions should be reviewed and re-aligned to fit the reform programme (Udofia and Uko, 2016). Such policy is believed to motivate the learner, lift some students higher standards, help increase the national productivity and contribute to the restoration of our global competitiveness. Demanding the absolute best from students, while keeping the material meaningful is one way to help improve work ethics. As student's achievement and assessment are directly connected, a substantial new approach to assessment was called for as a major premise of this higher education reform movement. This approach indicated a transition to learner-centered environments with frequent feedback for learners, lecturers, and institutions to make improvements (Huba and Freed, 2000).

Different approaches can be used in ensuring that students can reach a deeper and more personalized approach to learning Mathematics, reduces cramming of facts, infuses reading culture into students, assess skill of knowing where to find information and assesses the highest level of educational objectives (Nilson, 2003). One of such approach which has not been properly explored in Nigeria is Triangulation of assessment as a method and writing to learn as assessment for learning tool. In order to ensure that the triangulation method used for instruction and assessment are themselves free from unfairness, the analysis of their efficacy need to be conducted. This is not practiced in Nigeria now. The foregoing background information therefore constitute the theoretical rationale to examine the effects of writing to learn Mathematics in form of writing to demonstrate learning, performance – based assessment tools, triangulation assessment methods in form of reciprocity with peer, and project or opened time-based assessment methods on student's academic achievement, Self-concept, and interest as affective outcome in Mathematics in Colleges of Education in the South-South States of Nigeria.

➤ *Statement of the Problem*

Assessment practices in Nigeria's tertiary institutions, particularly Colleges of Education, have long been skewed toward a narrow focus on the cognitive domain. While lecturers are mandated to assess students across cognitive, affective, and psychomotor domains, actual practice typically emphasizes only the cognitive component, contributing 40% or more to terminal scores. This creates an incomplete and imbalanced view of student performance. Such disproportionate assessment has led to what many educators refer to as a "cobra effect"—a situation where attempts to solve one problem inadvertently worsen others. Over-reliance on traditional paper-and-pencil tests limits opportunities for creativity, encourages rote learning, and does not reflect real-life problem-solving situations. Consequently, students may develop negative attitudes toward learning and resort to unethical practices

such as examination malpractice in their pursuit of success. This lack of diversity in assessment methods has made it difficult to standardize learning outcomes across institutions and regions. It further undermines the goals of continuous assessment, which aims to holistically evaluate learners' knowledge, skills, and behaviors throughout the educational process (Carless, 2015; Herbert & Powell, 2016).

Assessment approaches that penalize innovation, ignore higher-order thinking skills, and overlook affective development are out of alignment with contemporary pedagogical goals. Moreover, national policies like Nigeria's Continuous Assessment Framework emphasize the need for multifaceted evaluation strategies. Yet, traditional assessments often fail to capture metacognitive skills, attitudes, and values which are key indicators of long-term learning and personal development (Sinwell, 2017). This gap presents a critical challenge in aligning assessment practices with the desired learning outcomes of 21st-century education. Therefore, this study responds to the urgent need for alternative assessment methods, such as writing-to-learn and triangulated approaches, which promote meaningful learning, self-reflection, and critical thinking. These strategies not only make learning visible but also engage students as active participants in the learning process, helping to reduce performance gaps, improve self-efficacy, and foster deeper conceptual understanding.

➤ *Purpose of the Study*

The primary purpose of this study is to investigate the effectiveness of an integrated intervention involving writing-to-learn tools and triangulated assessment methods in enhancing students' academic achievement and affective outcomes in mathematics education within Colleges of Education in Nigeria's South-South region.

• *Specifically, the Study Aims to:*

- ✓ Examine differences in academic achievement and affective outcomes between students assessed using writing-to-learn tools and those assessed with traditional methods.
- ✓ Determine the impact of triangulated assessment strategies (Reciprocity with peer, project-based evaluations) on student overall academic achievement and affective outcome of students with respect to learning Mathematics;
- ✓ Assess the combined effect of writing and triangulation on learning outcomes.
- ✓ Assessed how the nature of student's individual meta-cognitive functioning (Selfconcept, interest, attitudinal change, motivation and value) increased academic achievement in the learning of Mathematics;
- ✓ Assessed how the meta-cognitive functioning change (Self-concept, interest, attitudinal change, motivation and value) increased academic achievement in the learning of Mathematics;
- ✓ Determine which of the triangulation assessment method (Reciprocity or project-based) is most effective for increasing academic achievement and affective outcome of students in the learning of Mathematics;
- ✓ Determine which of the writing-to-learn assessment tools (writing to demonstrate knowledge or performance-based) is most effective for increasing academic achievement and affective outcome of students in the learning of Mathematics;
- ✓ Assesse the interaction effect of writing to learn as an assessment tool and triangulation assessment approach as method of evaluating academic achievement and affective outcome of students in the learning of Mathematics; and
- ✓ Explore gender-based differences in response to overall academic achievements and affective Outcome between students assessed using traditional methods and alternative assessments in the learning of Mathematics.

➤ *Research Questions*

To guide this study, the following research questions were raised.

- What mean difference exist between writing- to- learn assessment tool and traditional test method in overall academic achievement and affective outcome of students in the learning of Mathematics?
- What is the mean difference between triangulation approach as assessment method and traditional test method in overall academic achievement and affective outcome of students with respect to learning Mathematics?
- What mean difference exist in student's overall academic achievements and affective outcomes between the alternative assessment group and the traditional assessment group in Mathematics?
- How does the nature of student's individual meta-cognitive functioning (Self-concept, interest, attitudinal change, motivation and value) increased academic achievement in the learning of Mathematics?
- How does these meta-cognitive functioning change, (Self-concept, interest, attitudinal change, motivation and value) affect academic achievement of students in the learning of Mathematics?
- What is the mean difference between the assessment approach reciprocity with peer and project- based methods of evaluating and increase in academic achievement and affective outcome of students in the learning of Mathematics?
- What is the mean difference between the assessment tools, writing to demonstrate knowledge and performance- based and increase in academic achievement and affective outcome of students in the learning of Mathematics?
- What is the mean interaction effect of writing to learn as an assessment tool and triangulation assessment approach as method of evaluating academic achievement and affective outcome of students in the learning of Mathematics?
- What is the mean difference in male and female student's overall academic achievements and affective outcomes between the experimental and the control groups in the learning of Mathematics?

➤ *Hypotheses*

These null hypotheses were formulated to further guide this study:

- There exists no significant mean difference between writing to learn assessment tool and traditional test method in overall academic achievement and affective outcome of students the learning of Mathematics.
- There is no significant mean difference between triangulation approach as assessment method and traditional test in overall academic achievement and affective outcome of students in the learning of Mathematics.
- There exists no significant mean difference in overall academic achievements and affective outcomes between students in the alternative assessment and the traditional assessment groups in Mathematics.
- There is no significant relationship between the nature of student's individual metacognitive functioning (Self-concept, interest, attitudinal change, motivation and value), and increased academic achievement in the learning of Mathematics.
- There exists no significant relationship between the metacognitive functioning change (Self-concept, interest, attitudinal change, motivation and value) and increments in academic achievement of students in the learning of Mathematics.
- There exists no significant difference in the mean scores between the assessment approach reciprocity with peer and project-based techniques of evaluation and increase in academic achievement and affective outcome of students in the learning of Mathematics.
- There exists no significant in the mean score difference between the assessment tools, writing- to-demonstrate knowledge and performance- based tool and increase in academic achievement and affective outcome of students in the learning of Mathematics.
- There is no significant interaction effect of writing -to- learn assessment tool and the triangulation assessment approach of evaluating academic achievement and affective outcome of students in the learning of Mathematics.
- There is no significant mean difference in males and females student's overall academic achievements and affective outcomes between the experimental group and the control group in the learning of Mathematics

➤ *Significance of the Study*

Findings of this study hopefully, will be beneficial to many people through improving the poor academic achievement and affective outcome of Mathematics learners. These people include lecturers, students, curriculum developers, personnel of the ministry of education, examination bodies and the society in general.

This study offers several potential contributions to educational theory and practice. For mathematics lecturers, it introduces evidence-based methods for improving instruction and student engagement. By understanding how students use writing to process mathematical concepts, lecturers can refine their teaching approaches and identify conceptual gaps in real time. Writing to learn and triangulated assessment methods empower mathematics educators to implement more robust and learner-centered instructional strategies. These approaches shift the focus from simple recall to encouraging students to demonstrate conceptual understanding, fostering deeper learning. This theoretical framework proves highly beneficial given the need for empirical data to support the use of innovative assessment tools in measuring academic performance and affective outcomes in mathematics. The findings from this study are expected to support mathematics lecturers in identifying effective tools that can improve students' mathematical competence while addressing persistent learning challenges. Additionally, these insights will help instructors understand how students think, which can in turn inform and enhance instructional delivery. Importantly, the identification of which combination of methods or variable categories produces the strongest learning outcomes can guide educators seeking to adopt writing-to-learn and triangulation-based assessments. These methods also enable a holistic assessment of students, revealing their underlying abilities and performance across varied contexts. By establishing a framework for comparing assessment outcomes, this research promotes uniformity in student achievement reporting a critical need in Nigeria's diverse and evolving education system. Recent research continues to highlight the value of writing-to-learn and performance-based assessment in promoting metacognition and reflective thinking in STEM education.

From the learner's perspective, assessment serves not only as a measure of achievement but also as a platform for feedback and personal growth. Meaningful learning requires the integration of new knowledge into prior mental schemas, which is best achieved in challenging, performance-oriented environments. Writing to learn mathematics engages students in performing complex tasks that encourage them to connect concepts, evaluate their thinking, and reflect on how they approach learning. These metacognitive activities have been strongly linked to long-term knowledge retention and deeper mastery. At the student level, the use of writing and triangulated methods can foster reflective thinking, deepen subject understanding, and enhance motivation especially among female learners who often excel in linguistically oriented tasks. Writing provides students with the opportunity to articulate their understanding, challenge misconceptions, and personalize mathematical learning within real-life contexts. This findings hopefully will help the students to remove some of the societal apathy towards Mathematics and know that their achievement depends on their own active participations and interest not only on their lecturers. Thus, students will appreciate the need for their involvement in their Mathematics assessment and this may help them to acquire both Mathematical achievement and affective outcomes which may enhance capacity building and sustainable development. In order words the students were enabled towards achievement of national goal for Mathematics education.

This findings are beneficial to students as they were assessed using divers methods, this gave opportunity to those who are not good with the traditional method test to do better in their assessment scores. Furthermore, writing fosters discovery learning,

prompting students to express what they truly understand. When students articulate mathematical ideas through writing, they engage in internal dialogue—translating abstract concepts into personally meaningful representations. In this study, many participants reported uncovering misconceptions, deepening their understanding, and relating mathematics to everyday life. Students may also find this research empowering, as it gives them the opportunity to **express** their views about how they learn best thereby making their learning more visible and authentic. Writing, in this sense, becomes a student voice mechanism, which is increasingly valued in learner-centered education.

This research may also inform policymakers and curriculum developers, providing evidence on the strengths and limitations of current assessment practices. The findings could guide reforms that align with contemporary teaching goals such as inclusive and formative assessment strategies and promote a culture of lifelong learning in tertiary mathematics education. Furthermore, results from this study may aid course designers in selecting appropriate tools that align with assessment goals and learner needs. The findings can serve as a model for standardizing student evaluations across diverse institutions, addressing the comparability issues that currently challenge Nigeria's education system.

Moreover, adopting alternative assessment methods such as triangulation has the potential to reduce bias, mitigate exam-related anxiety, and enhance students' confidence and self-efficacy critical for equity in academic achievement. These methods align with the broader aim of education to advance social development, ensuring all learners have the opportunity to contribute meaningfully to society. This research can inform decision-makers about the limitations of current assessment frameworks and offer alternatives that are more inclusive, equitable, and aligned with global best practices. It advocates for moving beyond rote learning and embracing assessment for learning, an approach that has proven effective in cultivating metacognitive skills and long-term mastery.

The findings hopefully will be of significant benefit to examination bodies in the sense that, scores derived through this tools and methods are standardized, therefore the challenge of transformation of continuous assessment scores before incorporating into the final assessment scores by examination bodies will be solved. These examination bodies, West African Examinations Council (WAEC), Joint Admissions and Matriculation Board (JAMB), National Examinations Council (NECO), and other, could effectively use alternative assessment tools and methods in this study, in administering their examinations. It may also help them to ensure that all students are properly and holistically examined, with parallel and equal opportunity to perform in the three domains of learning. With the application of this alternative assessment tools and methods in obtaining continuous assessment scores (CAS) by schools, the findings will hopefully provide parents, students and lecturers with clear information about the performance of individual student as measured by a common/equivalent or national standard. Through this approach, quality assurance will be guaranteed in Nigeria testing enterprise.

This research fills a critical gap in the literature by providing empirical data on the use of writing-to-learn and triangulated assessment methods in Nigerian tertiary institutions. It contributes to ongoing scholarly discussions on innovative pedagogies and offers a practical framework for rethinking how learning is assessed, especially in STEM-related fields. As the labor market increasingly demands higher-order thinking and metacognitive skills, this study adds to the body of knowledge necessary for educators and curriculum designers to align assessments with real-world competencies. It also helps institutions gauge how well students meet academic standards and what interventions are needed to close performance gaps. Finally, the research may serve as a foundation for further inquiry into writing to learn, alternative assessments, and the implications for female mathematics learners, whose outcomes have shown consistent improvement when such strategies are applied.

➤ *Scope of the Study*

There are many alternative assessment tools and methods. This study sought to empirically investigate the use of some of these tools and methods in assessing the effect on students academic achievement and affective outcome in Mathematics. The approach involved using writing to demonstrate learning and performance- based assessment tools and reciprocity with peer, project or open-time for the triangulation method. The robustness in approaching the assessment for learning and sustainable assessment approach of evaluation in Colleges of Education, informed the choice of these methods. The content of assessment covered basic general Mathematics iv topics only, which are variation, linear, simultaneous and quadratic equations and statistics. The year two topics was chosen because it forms part of the basic foundations for learning any other concept in Mathematics and Mathematics related courses. Also this algebra and statistics are identified as difficult aspect of Mathematics due to their abstract nature. The items that were included in the test were restricted to the application, analysis, evaluation and creation level of cognitive domain of the Blooms revised taxonomy. This restriction is due to the level of Mathematics competence (abilities) expected of learners in the year two level of education as indicated in the minimum standard.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

Some key issues in assessment literature were reviewed in this chapter. The literature regarding assessment practiced in tertiary education were systematically reviewed and presented in this section. The literature review therefore, was done under conceptual, theoretical and empirical frameworks and Summary of reviewed Literature.

A. Conceptual Framework

This section presents foundational perspectives related to assessment in higher education, particularly the shift from traditional methods to alternative assessments that emphasize learning rather than testing. It explores key ideas surrounding writing as a learning tool, the writing-to-learn movement, and triangulation assessment approaches. The review spelled out the purposes of assessment, current modes of assessments, insight into the current practice of assessments, the impact on learning outcomes, and the characteristics of feedback were provided.

➤ *Concept of Assessment, Students Academic Achievement and Affective Outcomes*

Educational assessment plays a critical role in facilitating feedback that informs teaching and enhances student learning. Assessing learners' progress yields objective evidence crucial for informed decision-making within the educational sphere. Specifically, in Mathematics education, assessment can significantly enhance learning and stimulate student interest in the subject. Modern assessment practices focus not only on evaluating performance but also on nurturing metacognition and student engagement (Boud, Dawson, & Bearman, 2016). To effectively evaluate the evolving educational system in Nigeria, continuous assessment has been instituted as a policy across all educational levels. The implementation of Continuous Assessment (CA) policies across educational tiers in Nigeria, aims to measure students' development across cognitive, affective, and psychomotor domains, employing diverse assessment tools to evaluate various learning components.

• *Definitions of Assessment*

Assessment is broadly defined as the systematic process of gathering, analyzing, and interpreting evidence to determine student learning and make informed educational decisions (Brookhart, 2013). Scholars emphasize that assessment should encompass a variety of tools and formats to ensure fair judgment and to improve teaching and learning (Carless et al., 2014). Assessment as learning—a formative approach—is particularly useful in mathematics education as it encourages students to evaluate their own understanding and thinking processes. This perspective aligns with feedback-based and learner-centered education reforms (Nicol, Thomson, & Breslin, 2014). According to Sadler (2005), assessment involves forming judgments about the quality and extent of student achievement, thereby inferring the learning that has occurred. Brown (2004) describes assessment as interpreting information about student performance collected through diverse methods. Taras (2005) emphasizes that assessment yields either comparative or numerical ratings based on specific, weighted goals. Nguon (2013) defines assessment as gathering and discussing information from multiple sources to understand what students know and can do, ultimately using the results to improve learning. Assessment encompasses the process of collecting and interpreting data to make informed judgments about a person's, object's, or event's quality. It involves gathering information to facilitate decision-making regarding educational outcomes (Arhin, 2015). Assessment focuses on learning, teaching, and outcomes, providing insights to enhance both (Ezenwa-Nebife, 2014). It represents an interactive process between students and educators, offering feedback that informs teaching strategies and aids students in improving their learning and study habits (Bassey and Idaka, 2007).

A robust assessment program necessitates the use of various instruments, each designed to uncover unique aspects of student learning outcomes and the achievement of program objectives. Assessment is an ongoing endeavor aimed at comprehending and enhancing student learning (Barnett, 2007). According to Reilley, (2007), it entails articulating expectations, establishing appropriate criteria and standards for learning quality, systematically collecting, analyzing, and interpreting evidence to gauge performance against these benchmarks, and utilizing the findings to document, explain, and improve performance.

• *The Role of Assessment in Education*

Assessment serves as an indispensable tool in educational settings, guiding teaching and learning activities. It provides accurate information that facilitates informed decision-making. Regardless of the teacher's proficiency, the students' intelligence, or the adequacy of instructional materials, the absence of effective assessment mechanisms can render teaching efforts ineffective (Ezenwa-Nebife, 2014). Ajuonuma (2006) highlights that assessment is a process of gathering data to inform educational decisions, ensuring that teaching strategies align with student needs. The definitions of assessment vary among scholars due to differing perspectives. However, assessments generally serve two primary purposes: summative and formative. Summative assessments aim to evaluate student learning at the conclusion of an instructional period, often through tests or final projects, to determine if educational goals have been met as defined by Taras (2005) and Brown (2004). Conversely, formative assessments are conducted during the learning process, providing ongoing feedback that can be used by instructors to improve their teaching and by students to enhance their learning as preferred by Huba and Freed (2007) and Sadler (2005).

Beyond these purposes, assessment is a fundamental tool to support student learning as asserted by Leathwood (2005) and Ajuonuma (2006). Boud and Falchikov (2005) and Anikweze (2005) claim that, effective assessment methods should promote and

foster student learning rather than merely measure it. Assessment should not only aim to fulfill immediate course objectives but also build a foundation for students to engage in self-assessment throughout their academic and professional lives. Boud (2000) further suggests that assessment should not only aim to fulfil the immediate goals of a course or program, but also build a foundation for students to adopt their own assessment within their academic life and for their lifetime. As literature about assessment indicates, the phrase ‘assessment of learning’ is equated with summative assessment while ‘assessment for learning’ equates to formative assessment (McDowell, Wakelin, Montgomery and King, 2011; Stiggins, 2002; Yorke, 2003). These phrases and terms will be interchangeably utilized in this study.

- *Assessment of Learning Versus Assessment for Learning*

Assessment serves multiple essential functions in education, with varying interpretations depending on its focus. Some scholars emphasize summative purposes, which involve evaluating students’ academic progress and certifying their achievements. Others highlight formative assessment, which emphasizes learning support and developmental feedback. Assessment plays a dual role in educational systems it can either serve as a summative measure of learning outcomes or as a formative tool that enhances the learning process. Scholars conceptualize assessment in different ways, reflecting divergent philosophical and pedagogical orientations. For instance, Carless, Salter, Yang, and Lam (2016) explain that assessment is not solely about certifying learning but also about providing learners with feedback that fosters continued growth and reflection.

The distinction between summative and formative assessment remains critical in contemporary educational discourse. Summative assessment, according to Winstone & Boud (2020), typically involves the evaluation of learning at the end of an instructional unit and is primarily used to assign grades or determine certification. This approach emphasizes the measurement of student achievement and often neglects the learning process itself. Winstone and Boud (2020) assert that assessment must go beyond assigning grades; it should involve gathering evidence of learning to inform both teaching and continuous improvement. Summative assessment is typically used to certify students’ knowledge at the end of instruction, but it can inadvertently limit deep learning by focusing excessively on grades and outcomes (Bearman, Dawson, & Boud, 2020). This high-stakes nature often discourages creativity and promotes surface learning strategies. Though summative assessment remains crucial for academic validation and progression, it must be balanced with authentic, learner-centered strategies. Braun (2014) advocates for the use of real-world tasks—such as project-based learning, reflective journals, and portfolios—even in summative settings. These methods encourage students to think critically and demonstrate understanding beyond rote recall.

In contrast, Boud and Molloy (2013) and Carless et al. (2016) advocate for formative assessment that supports students during learning. They argue that when learners are provided with actionable feedback, their ability to self-regulate and reflect improves significantly. Nicol (2019) reinforces this by viewing feedback not as a one-way transmission but as a core process within the learner’s internal cognitive framework. Conversely, formative assessment is oriented toward learning improvement. Nicol (2019) and Boud and Molloy (2013) emphasize that feedback-rich, formative assessment strategies can significantly improve self-regulation, motivation, and engagement. This kind of assessment focuses on identifying learning gaps and offering actionable feedback that supports students’ progression toward mastery. A key concern with summative assessment is its potential to narrow learning goals. As Bearman, Dawson, and Boud (2020) argue, a high-stakes summative environment often prioritizes surface learning strategies over deep understanding. This is echoed in studies highlighting that students may develop test-taking skills at the expense of meaningful learning when emphasis is placed solely on performance metrics (Evans et al., 2020). However, researchers suggest that summative assessment need not be inherently detrimental if integrated with authentic, learner-centered approaches. For instance, Braun (2014) advocates for summative assessments that include journals, portfolios, and projects, which allow learners to demonstrate understanding in practical and reflective ways.

The shift from assessment of learning to assessment for learning (AfL) marks a pedagogical reorientation from evaluation to facilitation. Black and Wiliam (2018) highlight that AfL should be embedded into classroom practices to yield real-time insights into student understanding. This enables educators to tailor instruction and students to adjust their learning strategies accordingly. Effective formative assessment provides students with timely, descriptive feedback that bridges the gap between current and desired performance. According to Henderson, Ryan, and Phillips (2019), feedback is only valuable if students understand and act on it—thus, the timing, clarity, and relevance of feedback are paramount. Moreover, Carless and Boud (2018) promote the idea of “feedback literacy”—the ability of students to interpret and use feedback productively—as a key outcome of assessment for learning. This view positions students not as passive recipients of grades but as active participants in the evaluation process. Formative assessment, or assessment for learning (AfL), allows for ongoing feedback and learning adjustments.

According to Henderson, Ryan, and Phillips (2019), timely and specific feedback encourages students to bridge gaps between current performance and learning goals. This feedback must be clear, relevant, and delivered in a way that encourages learners to take ownership of their progress. An essential aspect of AfL is the concept of feedback literacy, which refers to a student’s ability to interpret and use feedback effectively. Carless and Boud (2018) suggest that developing this literacy empowers learners to become active participants in the feedback process, ultimately leading to better academic performance. Formative assessment is increasingly recognized for its role in promoting deeper learning and self-regulation. Gezer et al. (2021) found that formative assessments in mathematics classrooms positively correlate with improved student performance, particularly among lower-achieving students. This

approach encourages active student participation and continuous feedback, essential for developing critical thinking and problem-solving skills.

The second objective of assessment for learning (AfL) emphasizes the enhancement of students' learning. The primary goal of AfL is to provide feedback that can support and accelerate both teaching and learning. According to recent studies, formative assessment coupled with constructive feedback is fundamental in fostering self-regulated learning habits in students (Clark, 2015; Panadero et al., 2018). One of the key functions of AfL is the continuous provision of actionable feedback that closes the gap between a student's current level of performance and the expected learning outcomes, rather than merely assigning grades or scores (Panadero & Alonso-Tapia, 2014). In line with this, several scholars emphasize that formative assessment encourages learning by enabling students to engage with real-time feedback derived from activities such as discussions, observations, and other classroom interactions (Black & Wiliam, 2018; Carless, 2015). This type of feedback is aligned with the contemporary educational model that places importance on the knowledge construction process. Feedback plays a crucial role when it is timely and detailed, ensuring students can effectively incorporate it into their future academic efforts (Evans, 2016).

Moreover, AfL is now widely adopted in global education policies, signaling a transformation from assessment for ranking to assessment for learning. According to Willis (2014), formative assessment represents a cultural shift that prioritizes individual progress and deeper understanding over performance metrics. Gipps and Stobart (2016) stress that communicating learning goals and success criteria allows students to self-evaluate, enhancing autonomy and long-term motivation. Assessment for learning is broadly defined as any strategy designed to improve student learning. Chalmers et al. (2014) report that although many students still gravitate towards summative assessment due to its high-stakes nature, educational institutions are increasingly embedding formative assessment into their curricula. An illustrative case is the Organization for Economic Co-operation and Development (OECD, 2018), which endorses formative assessment as a tool that equips students with lifelong learning skills essential in today's fast-changing knowledge economy.

From an economic and pedagogical standpoint, formative assessment is tightly connected to concepts like lifelong learning and the knowledge society, as it promises not just immediate academic gains but long-term competence across learner populations (Kennedy, Fok, & Chan, 2016; Looney, 2014). Consequently, education stakeholders continue to support formative assessment practices. Burns (updated by Andrade & Brookhart, 2019) reiterated that assessment serves three main purposes: helping educators modify instruction, giving insights into student understanding, and reporting progress to stakeholders. While summative assessments serve only the third function, formative assessments fulfill all three, thus offering a broader educational impact. Evidence suggests that assessments that focus on rote learning may encourage shallow engagement, while those that require comprehension and real-world application foster deeper learning strategies (Weurlander et al., 2018).

Additionally, summative assessment data can be repurposed formatively by providing feedback that facilitates student improvement (Brookhart, 2017). Classroom-based formative assessment, such as writing to learn strategies, is seamlessly integrated into everyday learning tasks. Educators frequently provide feedback on journal entries, portfolios, quizzes, and project work. When this feedback highlights specific areas for improvement regardless of whether it includes grades it fosters better student performance. Meaningful commentary on written assessments equips students to identify their weaknesses and take action for improvement (Wiliam, 2020). In the context of Mathematics education, such challenges can be addressed by incorporating formative assessment tools like writing exercises, which not only assess learning but also support instruction. Formative assessment, therefore, functions as an integral part of the teaching-learning process. It supports a feedback-rich environment where students receive timely responses to their work, enabling them to bridge the gap between their current performance and desired learning outcomes (Andrade & Heritage, 2018). Feedback within AfL must be continuous and actionable, allowing learners sufficient time to reflect, revise, and apply suggested improvements. As Carless and Winstone (2020) emphasized, effective feedback should not just be informative but also feed forward—guiding students on how to improve future performance.

Several scholars have stressed the importance of AfL through different lenses. Price et al. (2019) reiterated that formative assessment includes the use of diverse data collection strategies—like observation, discussion, and self-evaluation—to give students insight into their learning journey. This aligns with the standard educational model which asserts that assessment should not solely be a summative measurement of knowledge, but rather a tool that helps learners acquire and apply that knowledge effectively (Wiliam, 2016). Moreover, Boud and Molloy (2013) warned that formative assessment could lose its effectiveness if feedback is delayed or inconsistent. For feedback to catalyze learning, it must be embedded within the instructional timeline so students have the opportunity to act on it. Similarly, the OECD (2015) emphasized that teachers who implement formative assessment strategies are more likely to cultivate students' "learning to learn" competencies—skills critical in the rapidly evolving knowledge-based economy.

Formative assessment is also linked to the concepts of lifelong learning and adaptability in an increasingly digital and information-centric society. As noted by O'Leary, Sloane, and Watson (2020), formative approaches encourage reflection, problem-solving, and critical thinking—attributes essential for students navigating uncertain futures. Kennedy, Chan, Fok, and Yu (2014) also supported the implementation of formative practices, citing their effectiveness in enhancing learner engagement and conceptual understanding across disciplines. Burns (reinterpreted by Avargil, Herscovitz, & Dori, 2018) identified three key purposes of

assessment: first, to provide teachers with data for instructional modification; second, to give insights into student comprehension; and third, to serve as a formal record for reporting to parents or stakeholders. While summative assessment primarily addresses the third purpose, formative assessment addresses all three, offering a more comprehensive mechanism for improving both teaching and learning.

Further evidence supports that the design of assessment tasks significantly impacts the depth of student learning. According to Weurlander et al. (2015), tasks that target factual recall may lead students to adopt surface-level learning approaches. In contrast, assessments requiring interpretation, application, and integration of ideas promote deeper engagement. This underscores the importance of designing assessment tools that encourage analytical and reflective thinking. Dunn and Mulvenon (2016) posited that even summative assessments can be used formatively if their results are analyzed and communicated in a way that supports improvement. Writing to Learn (WTL), for example, is a powerful formative assessment technique embedded into regular classroom activities. Through journaling, portfolios, quizzes, and performance-based tasks, students are encouraged to reflect on their learning while simultaneously being assessed. Lecturers who employ WTL strategies often provide personalized feedback highlighting student strengths and areas needing growth. This targeted guidance, when coupled with opportunities for revision, has been shown to significantly enhance learning outcomes (Brookhart, 2017). Importantly, the success of WTL relies on the feedback loop—not merely assigning a score but fostering a developmental process where students engage critically with feedback and adjust their learning strategies accordingly.

The transformative nature of AfL lies in its ability to humanize the learning experience. Rather than viewing assessments as judgmental or punitive, students begin to see them as integral to their growth. As argued by Sadler (2014), feedback must be specific, clear, and focused on improvement, not merely evaluation. When students understand what success looks like and how to achieve it, their motivation and confidence increase. Furthermore, embedding formative assessment into instructional practice—especially through WTL in Mathematics—serves dual functions. It promotes cognitive engagement by requiring students to articulate their understanding in writing and it supports metacognitive development by encouraging reflection on learning processes. This makes WTL a valuable instructional and assessment tool, particularly in disciplines like Mathematics, where procedural knowledge often overshadows conceptual understanding (Braund & DeLuca, 2021). In summary, Assessment for Learning, particularly when executed through writing-based practices, fosters a supportive learning environment that encourages deep thinking, continuous improvement, and learner autonomy. The evidence strongly supports its integration into educational practice—not as a replacement for summative assessments, but as a complementary strategy that enhances the overall educational experience and better prepares students for real-world problem-solving and lifelong learning. In conclusion, while summative assessment remains a staple in academic institutions due to its administrative and certification purposes, the integration of formative strategies is essential for promoting deeper learning and self-regulation. Educational institutions are increasingly encouraged to balance both assessment types to support meaningful and sustainable learning experiences.

- *Assessment as Learning and Learning-Oriented Assessment*

Assessment as learning empowers students to engage actively in their learning processes, fostering self-evaluation and autonomy. Earl (2013) conceptualizes 'assessment as learning' as students' engagement in self-assessment, enabling them to monitor and direct their own learning journey. This approach aligns with constructivist theories, emphasizing the role of learners in constructing knowledge through active participation. Hall and Jones (2019) further elaborate that 'assessment as learning' involves designing assessment tasks that cultivate critical thinking and deepen students' understanding. Such tasks encourage learners to analyze, evaluate, and synthesize information, promoting higher-order cognitive skills essential for academic success. Boud and Falchikov (2015) draw from Mentkowski and Associates's philosophy, suggesting that "assessment as learning represents an attempt to create 'learning that lasts'." They argue that when teaching, learning, and assessment are coherently integrated throughout courses and programs, students achieve more meaningful and enduring learning outcomes. Building upon this, Carless (2018) introduces the concept of 'learning-oriented assessment' (LOA), aiming to bridge formative and summative assessments to promote productive student learning. According to Carless, LOA comprises three interrelated components: assessment tasks as learning tasks, student involvement in assessment, and the closing of feedback loops.

- *Assessment Tasks as Learning Tasks*

Designing assessment tasks that serve as learning opportunities is central to LOA. Carless (2018) emphasizes that tasks should be constructively aligned with curriculum objectives, encouraging deep learning approaches. Such alignment ensures that assessments not only evaluate student performance but also facilitate the development of critical skills and knowledge. Moreover, Keppell and Carless (2016) advocate for assessment designs that mirror real-world tasks and promote cooperative learning environments. This approach is evident in 'writing to learn' strategies within mathematics education, where students articulate their understanding through written explanations, fostering both comprehension and communication skills.

- *Alternative Assessment and Authentic Learning*

The evolution of assessment practices in higher education has led to the adoption of alternative assessments that emphasize authentic learning experiences. Wiggins (2010) argues that authentic assessments require students to perform tasks that reflect real-world challenges, such as conducting research, engaging in debates, or collaborating on projects. These alternative assessment methods, including portfolios, simulations, and problem-based learning, enhance deeper learning and foster critical thinking skills.

Cummings, Maddux, and Richmond (2014) highlight that such assessments encourage students to take ownership of their learning, promoting self-regulation and lifelong learning habits.

- *Student Involvement in Assessment*

Active student participation in the assessment process is a cornerstone of LOA. Carless (2018) posits that involving students in dialogues about assessment criteria and standards builds trust and transparency between educators and learners. This engagement enhances students' understanding of learning objectives and fosters a sense of responsibility for their academic progress. Orsmond, Merry, and Reiling (2015) support this view, noting that students who engage with assessment criteria and participate in peer assessments develop a deeper comprehension of quality work. Peer feedback serves as an interactive mechanism for students to exchange ideas, reflect on their performance, and develop self-regulation skills. Boud (2016) introduces the concept of 'sustainable assessment,' emphasizing the importance of self-assessment skills that students can carry into their professional lives. By developing the ability to evaluate their work critically, students become autonomous learners capable of adapting to various learning contexts.

- *Feedback and Student Learning*

Effective feedback is integral to student learning within the LOA framework. Carless (2018) asserts that timely and constructive feedback enables students to 'feed forward,' applying insights to future tasks. However, Wiliam (2014) cautions that feedback must be actionable and lead to improved performance to be truly effective. Gibbs and Simpson (2015) add that for feedback to enhance learning, students must engage with it actively, reflecting on the information provided and implementing necessary changes. Falchikov (2016) also highlights the value of peer feedback, noting that it complements instructor feedback and fosters a collaborative learning environment. In summary, 'writing to learn' in mathematics exemplifies the principles of LOA by integrating well-designed assessment tasks, encouraging student involvement, and providing effective feedback. This approach not only supports current learning objectives but also equips students with skills essential for lifelong learning and professional success.

- *Sustainable Assessment: Preparing Learners for Lifelong Learning*

Sustainable assessment is an evolving concept in educational discourse that emphasizes the development of learners' abilities to assess and regulate their own learning beyond formal education settings. Originally introduced by Boud (2000), sustainable assessment is defined as "assessment that meets the needs of the present without compromising the ability of students to meet their own future learning needs." This approach shifts the focus from traditional assessment methods that prioritize grading and certification to practices that foster lifelong learning skills. He emphasized that traditional assessment methods often focus solely on immediate academic performance, neglecting the preparation of students for lifelong learning and adaptability in their professional lives. Boud argued for an assessment approach that fosters continuous learning beyond formal education.

- *Core Principles of Sustainable Assessment*

Boud and Soler (2016) revisited the concept of sustainable assessment, highlighting several key principles:

- ✓ Development of Self-Assessment Skills: Encouraging students to engage in self-evaluation to foster independent learning.
- ✓ Emphasis on Learning Processes: Focusing on the learning journey rather than solely on outcomes.
- ✓ Integration of Feedback Mechanisms: Utilizing feedback not just for grading but as a tool for continuous improvement.

These principles aim to equip students with the skills necessary for self-regulation and continuous learning, which are essential in adapting to the ever-changing demands of the modern world.

- *Challenges in Implementing Sustainable Assessment*

Despite its potential benefits, implementing sustainable assessment practices poses several challenges. Institutional policies, time constraints, and existing educational cultures that prioritize summative assessments over formative ones can hinder the adoption of sustainable assessment strategies (Carless et al., 2011). Moreover, both educators and students may require additional support and training to effectively engage in self-assessment and peer feedback processes. Boud (2000) concluded that to achieve sustainable assessment, greater attention must be given to the effects of summative assessments, seeking ways to reform them to support lifelong learning.

- *The Role of Feedback in Sustainable Assessment*

Feedback plays a crucial role in sustainable assessment by facilitating students' ability to monitor and evaluate their own learning. Carless et al. (2011) emphasize the importance of dialogic feedback processes that involve students in discussions about learning, thereby enhancing their awareness of quality performance and developing their capacity for self-regulation. Such feedback mechanisms not only support current learning tasks but also prepare students for future learning challenges. Sustainable assessment represents a paradigm shift in educational assessment practices, focusing on developing learners' capacities for lifelong learning. By emphasizing self-assessment, continuous feedback, and the development of evaluative judgment, sustainable assessment prepares students to navigate the complexities of the modern world effectively. However, realizing the full potential of sustainable assessment requires concerted efforts to address implementation challenges and foster a culture that values continuous learning and self-improvement.

- *Affective Assessment*

In contemporary educational discourse, there is an increasing recognition of the necessity to balance the assessment of learning outcomes by encompassing all domains associated with behavioral changes, rather than focusing solely on cognitive achievements. Educators are encouraged to assess affective outcomes in learners, as this holistic approach equips students not only with academic competencies but also with the requisite knowledge, skills, attitudes, values, and psychosocial abilities essential for leading healthy, satisfying lives and maximizing the benefits of learning (Adetayo, 2014). The affective domain, integral to Bloom's taxonomy, emphasizes learning objectives that involve emotional responses, attitudes, and values. This domain is inherently more challenging to objectively analyze and assess, given that affective objectives range from simple attention to complex, internally consistent qualities of character and conscience (Inekwe, 2019). Despite these challenges, the educational process must address the assessment and measurement of students' abilities within this domain. The distinction between being "schooled" and being "educated" often underscores the neglect of affective development in favor of cognitive instruction.

Affective learning competencies are typically articulated through instructional objectives—statements delineating the new capabilities learners should acquire by the end of instruction. These objectives are specific, measurable, short-term, and observable student behaviors. They serve as foundational tools for constructing lessons and assessments aligned with overarching course or lesson goals (Inekwe, 2019). While affective learning is assessed and measured in educational settings, it is seldom utilized as a grading criterion. Behavioral objectives within the affective domain focus on observable behaviors, facilitating their translation into quantitative terms. The hierarchical structure of the affective domain includes:

- ✓ Receiving: Accepting, attending to, developing, recognizing.
- ✓ Responding: Completing, complying, cooperating, discussing, examining, obeying, responding.
- ✓ Valuing: Accepting, defending, devoting, pursuing, seeking.
- ✓ Organization: Codifying, discriminating, displaying, ordering, organizing, systematizing, weighing.
- ✓ Characterization: Internalizing, verifying.

In assessing learning competencies within the affective domain, several focal concepts are considered:

- *Attitudes*

Attitudes are mental predispositions to act, expressed through evaluations of entities with varying degrees of favor or disfavor. Individuals possess attitudes directed toward objects, people, or institutions, often linked to mental categories. These attitudes comprise four components:

- ✓ Cognitions: Beliefs, theories, expectations, cause-and-effect beliefs, and perceptions related to the focal point.
- ✓ Affect: Feelings associated with the focal object, such as fear, liking, or anger.
- ✓ Behavioral Intentions: Goals, aspirations, and expected responses toward the attitude object.
- ✓ Evaluation: Central component involving judgments of goodness or badness toward an object.

Attitudes significantly influence behavior, serving as frameworks for interpreting and responding to individuals, concepts, or ideas within social communities (Petrosyan et al., 2005).

- *Motivation*

Motivation encompasses the reasons for engaging in specific behaviors, including basic needs, goals, or ideals deemed desirable. It pertains to the initiation, direction, intensity, and persistence of human behavior. In educational contexts, motivation affects how students learn and their engagement with subject matter. It directs behavior toward particular goals and exists in two forms:

- ✓ Intrinsic Motivation: Derives from internal satisfaction or moral significance associated with learning.
- ✓ Extrinsic Motivation: Stems from external factors compelling a student to act.

- *Self-Concept*

Self-concept refers to an individual's perception of their capabilities to perform tasks or achieve goals. It encompasses beliefs about one's ability to execute actions required to manage prospective situations. Self-efficacy, a component of self-concept, influences motivation and behavior. Research indicates that overestimating one's efficacy can negatively impact motivation, while underestimating it may enhance the drive to study (McMunn, 2011). Teaching and learning are complementary activities occurring formally in schools and informally at home or within the community. Teaching involves actions by educators to help students acquire and retain knowledge, attitudes, and skills. Learning is associated with behavioral changes across cognitive, affective, and psychomotor domains. Educators play a crucial role in determining what, how, and when to teach, as well as evaluating the effectiveness of the teaching-learning process through classroom assessments (Erinosho & Badru, 2000). Traditional assessment practices in schools have predominantly utilized paper-and-pencil tests, emphasizing test results and examinations. Consequently, attention has been focused on assessing cognitive variables at the end of formal instruction, often through achievement tests in specific subject areas. However, educationists and evaluators acknowledge the importance of affective behaviors in education and

evaluation. Despite this, deliberate attempts to assess affective outcomes are often lacking, with the attainment of learners viewed as a function of their entire personality (Adetayo, 2014).

Affective variables are crucial in the educational process. Students' attitudes toward learning significantly influence the extent of their engagement and pursuit of knowledge. Values related to truthfulness and integrity shape daily conduct, and self-esteem impacts nearly all aspects of student behavior. Despite their importance, few educators explicitly focus on influencing students' attitudes and values, and even fewer assess students' affective status systematically. Observations of students' emotional states are often anecdotal, lacking systematic assessment (McMunn, 2011). Students capable of producing outstanding work but who perceive themselves as inadequate may avoid challenges, highlighting the significance of affective variables. Specialists in assessment increasingly recognize affective factors as sometimes more critical than cognitive variables. Individuals with moderate intellectual abilities may succeed due to high motivation and diligence, while highly capable individuals may falter due to low self-worth. The profound impact of affective status on behavior underscores its importance in educational settings, particularly in higher education, where students develop motivation, resilience, self-awareness, and moral values essential for a fulfilling life (Stiggins, 2005).

- *School Affective Behaviours*

In educational settings, particularly within higher institutions and specifically college classrooms, it is imperative to instill fundamental human values. Lecturers play a crucial role in evaluating whether students exhibit these essential values and attitudes. Various attitudinal behaviours can be targeted through instructional strategies. Anderson and Bourke (2000) identified several dimensions of classroom affect that directly influence students' motivation to learn. These dimensions include attitudes, school-related values, academic self-efficacy, interests, academic aspirations, motivation, and evaluation or assessment anxiety. Recognizing and evaluating these behaviours enable educators to foster environments conducive to both academic and personal growth.

- *Attitudes and School-Related Values*

Attitudes refer to individuals' predispositions to respond favorably or unfavorably towards specific ideas, objects, or persons. In the classroom, students' attitudes towards subjects, peers, and instructors can profoundly impact their engagement and achievement. Positive attitudes are often linked to increased motivation and better academic performance.

School-related values, on the other hand, are the principles and beliefs that students hold regarding education and its significance. These values, such as the importance of honesty, responsibility, and perseverance, are integral to students' academic integrity and commitment. Educators play a crucial role in instilling and reinforcing these values through curriculum design and classroom interactions.

- *Academic Self-Efficacy*

Academic self-efficacy is the belief in one's capabilities to organize and execute actions required to manage prospective academic situations. Students with high self-efficacy are more likely to embrace challenging tasks, persist in the face of difficulties, and achieve higher academic success. Research indicates that self-efficacy is influenced by various factors, including past experiences, social persuasion, and physiological states. Educators can enhance students' self-efficacy by providing constructive feedback, setting achievable goals, and creating supportive learning environments.

- *Interests and Academic Aspirations*

Students' interests in specific subjects or activities can significantly affect their motivation and engagement levels. When students are interested in what they are learning, they are more likely to invest time and effort, leading to deeper understanding and retention of information. Academic aspirations, which refer to students' educational goals and ambitions, are closely tied to their interests and self-concept. Educators can nurture students' interests and aspirations by offering diverse learning opportunities and connecting academic content to real-world applications.

- *Evaluation Anxiety*

Evaluation or test anxiety is a psychological condition characterized by excessive stress and worry about academic assessments. High levels of evaluation anxiety can impair students' performance, reduce their confidence, and hinder their overall academic progress. To mitigate evaluation anxiety, educators should employ fair and transparent assessment methods, provide practice opportunities, and teach stress-reduction techniques. Creating a classroom atmosphere that emphasizes learning over competition can also alleviate anxiety.

- *Performance Assessment and Affective Behaviours*

Performance assessments, which require students to demonstrate their knowledge and skills through practical tasks, offer valuable insights into students' affective behaviours. These assessments can reveal students' problem-solving abilities, creativity, collaboration skills, and perseverance. By integrating performance assessments into the curriculum, educators can evaluate both cognitive and affective domains, leading to a more comprehensive understanding of student learning. Assessing affective behaviours in educational settings is essential for fostering well-rounded individuals equipped with the necessary skills and attitudes for lifelong learning. Educators must recognize the significance of affective domains and implement strategies to evaluate and enhance these

behaviours. By doing so, they contribute to the development of students who are not only academically competent but also emotionally intelligent and socially responsible.

- *Importance of Affective Outcome in Learning*

Students' daily conduct is largely shaped by personal values such as honesty, integrity, and a healthy self-esteem, which exert a substantial influence on nearly every aspect of their academic and social lives. There is broad agreement among education scholars that students' emotional and attitudinal disposition—what is commonly referred to as their affective status—should be a critical concern for educators (Anderson, Graham & Thomas, 2016). Yet in practice, only a limited number of classroom instructors make a deliberate effort to foster or assess these affective attributes in their students. This gap is particularly noticeable among lecturers in higher education, who often focus exclusively on cognitive and skill-based outcomes, assuming that emotional and attitudinal development lies outside their educational responsibilities (Mensah, Okyere & Kuranchie, 2019). It is common for teachers to form observational impressions about their students' moods or behaviour, such as assuming a student may be withdrawn or anxious. However, it is quite rare to find educators who systematically document or assess these affective traits. According to recent research, comprehensive tools or procedures to assess affective states among students are still not widely utilized in most classrooms, especially at the tertiary level (Adeyemo & Olanrewaju, 2020). This lack of attention to affective development results in a missed opportunity to holistically educate students.

Many capable students fail to realize their full potential not because of cognitive deficits, but because of negative self-perceptions or a lack of motivation. For instance, a student who is highly competent in essay writing but perceives themselves as poor writers is unlikely to engage in writing unless compelled. In contrast, there are numerous cases of individuals who may not be naturally gifted but succeed due to high levels of self-motivation, resilience, and a growth mindset (Yildiz & Kula, 2015). Conversely, highly intelligent individuals may fail to act on their potential simply because of feelings of self-doubt or low self-worth. These examples demonstrate that affective attributes such as motivation, perseverance, and personal values are critical determinants of success, not just in school but also in life. In colleges and universities, where students are expected to cultivate their identities and values, the significance of affective development cannot be overstated (Oduwaiye, Amadi & Lawal, 2021).

- *The Role of Affective Learning Outcomes*

Affective learning components such as attitudes, values, self-concept, and motivation are integral to the development of student behaviour and future aspirations. These variables do not operate in isolation; they interact closely with cognitive development to influence how learners approach and value education (Güvendir & Özkanal, 2020). Students who exhibit a positive attitude toward education tend to display a stronger willingness to continue learning over time. Thus, the development of positive affective characteristics can set the foundation for lifelong learning behaviours. Beliefs and values acquired during one's academic life tend to have lasting implications. For instance, a student who internalizes the importance of health and wellness during college is more likely to maintain health-conscious behaviours throughout adulthood. Similarly, someone who learns to value interpersonal respect and community over material gain is more likely to act in socially responsible ways (Uka, 2020).

In light of these long-term implications, it becomes imperative for educators, particularly in tertiary institutions, to pay close attention to the affective domain of learning. Lecturers must recognize that the current emotional and attitudinal state of a learner is not only a predictor of academic achievement but also a critical determinant of the student's future behaviour and personal development (Chen, Li & Zhao, 2019). As such, incorporating affective assessment tools and intentionally designing instruction to address this domain is essential for a balanced and holistic educational experience.

- *School Affective Behaviours*

Educational institutions, particularly at the tertiary level, have a responsibility to inculcate core human values within students. Consequently, lecturers are expected to evaluate whether students display key affective traits and value systems essential for academic and personal development. Various types of attitudinal behaviour fall within the spectrum of a lecturer's instructional responsibilities (Kurebwa, 2015). Anderson and Bourke (2014), along with Owolabi and Olaseinde-Williams (2017), identified several crucial dimensions of classroom affect: student attitudes, value systems tied to school life, academic self-efficacy, interest in subjects, academic goals, personal motivation, and test or assessment-related anxiety. Each of these factors plays a pivotal role in shaping students' motivation to engage and learn meaningfully.

Attitudes, often defined as favourable or unfavourable dispositions toward people, activities, or concepts, can shift easily, especially in young adults. They influence how students perceive school subjects or instructional methods. School-related values reflect what learners believe to be desirable or important—these may include the perceived benefits of education, belief in hard work, or the quality of relationships with lecturers, which encompass honesty, trust, and mutual respect (Anderson & Bourke, 2014).

Values, beyond personal belief systems, function as tools for intergenerational cultural transmission. In a school context, this may involve beliefs such as valuing education as a key to a successful future, or appreciating integrity in academic relationships. Academic self-concept, on the other hand, is among the most directly school-related affective traits. It refers to a student's self-evaluation of their potential to perform and succeed academically. According to Stiggins (2016), it is critical that lecturers help students link their academic efforts with their resulting success. When students perceive academic achievement as being within their

control, they are more inclined to invest effort and persist toward goals. This internalized sense of academic control can significantly improve academic outcomes.

Student interests also play a crucial role in motivation. These interests are defined as preferences for specific tasks, ideas, or problem-solving approaches. For instance, a student might show enthusiasm for digital simulations but display disinterest in traditional oral interviews. Identifying such preferences enables lecturers to design instruction tailored to specific student needs. Academic aspiration, another vital element, refers to students' long-term commitment to educational engagement. When students perceive learning as meaningful and growth-oriented, they are more likely to remain invested in their studies. Conversely, if students feel disengaged or unchallenged, they may choose to mentally or physically disengage from the learning process.

Assessment anxiety is another critical dimension that educators must manage. Lecturers must create an environment where students approach assessment with confidence, rather than fear or anxiety. This includes preparing students to manage test stress and promoting perceptions of fairness in grading and feedback. Beyond internal assessments, students must also be prepared to handle high-stakes exams and academic competitions outside their institutions. Understanding and addressing the full spectrum of classroom affect—attitudes, motivation, values, interests, and self-concept—can help build confident learners who actively seek knowledge and persist in the face of academic challenges.

One effective method for fostering positive affective behaviour is the use of performance-based assessments such as writing to learn strategies in Mathematics. This approach asks students to demonstrate mastery of concepts through applied, practical tasks rather than rote memorization. It integrates cognitive and affective engagement by enabling students to use prior knowledge, synthesize ideas, and apply what they have learned to solve real-world problems (Etsey, 2016). Through this method, students are not merely tested on content retention but are evaluated based on their ability to interpret, communicate, and apply concepts in diverse contexts. Back and Hwang (2017) argue that performance-based assessment in education not only strengthens learning outcomes but also encourages creativity and intellectual exploration among students. Their research showed that such assessments contribute to growth in students' academic achievement, cognitive engagement, creativity, and problem-solving ability. Writing to learn Mathematics allows for classroom dialogues guided by the teacher, which in turn help generate multiple perspectives on a given problem. These peer and teacher-led discussions strengthen comprehension and enrich problem-solving strategies.

Furthermore, students taught using writing to learn strategies have shown increased motivation and interest in Mathematics. The structured, step-by-step nature of such instruction enables them to navigate problems more effectively, thereby improving their confidence and intrinsic interest in the subject. This method also enhances the capacity for self-reflection, allowing students to evaluate their learning processes and develop metacognitive skills essential for lifelong learning. In summary, school affective behaviours must be carefully cultivated and assessed alongside cognitive performance. This ensures a balanced, holistic educational approach that empowers students not just intellectually but also socially and emotionally.

➤ *Writing to Learn: Enhancing Student Achievement, Self-Concept, and Interest (Affective Outcomes)*

The Writing Across the Curriculum (WAC) initiative, which began in the late 1800s, has significantly influenced higher education by promoting the integration of writing into various academic disciplines. In 1989, the National Council of Teachers of Mathematics (NCTM) identified mathematical communication as a key goal in their Curriculum and Evaluation Standards. This emphasis encouraged students across all educational levels to articulate their mathematical thinking through discussion and writing, fostering a deeper understanding of mathematical concepts. By 2000, NCTM's Principles and Standards further reinforced the importance of writing in mathematics classrooms, advocating for environments where students could express and refine their mathematical ideas both verbally and in written form. (NCTM, 2000). This document emphasized writing in Mathematics classrooms so that students could test their understandings on the basis of shared knowledge. Students who engage in Mathematical communication (via writing, speaking, reading, and listening) have the twofold benefit of communicating to learn Mathematics and learning to communicate Mathematically (NCTM, 2000; Pugalee, 2004). As students mature, their Mathematics communication should also progress. Students at increasing developmental levels acquire increasing Mathematical abilities and knowledge; and in a similar manner, lecturers should build on student's Mathematics writing abilities as they progress through school (NCTM, 2000).

Writing serves as a powerful tool for learning, particularly in mathematics. Many students perceive success in mathematics as merely obtaining correct answers or memorizing facts (Van Dyke, Malloy and Stallings, 2014). However, writing helps them recognize that mathematics involves complex processes as affirmed by Knox, (2017), and connections to real-life experiences (Braun, Diaz and Dykes, 2015). Through writing, students can reflect on their problem-solving strategies, clarify their understanding, and develop a personal narrative that connects mathematical concepts to their own lives (Baxter, Woodward and Olson, 2005). This reflective practice enables students to identify gaps in their knowledge and fosters a deeper engagement with the subject matter. (Palmer, 2018). The use of writing-to-learn (WTL) strategies in mathematics has gained global attention. Unlike traditional assessment, WTL tasks require students to articulate, reflect, and explain their problem-solving processes. This engages students cognitively and emotionally, fostering deep learning (Palma, 2018; Kenney, Shoffner & Norris, 2014). WTL activities include reflective journals, mathographies, and problem solution narratives. These tools help learners construct knowledge actively, enhance mathematical reasoning, and improve affective traits such as confidence and interest (Nilson, 2014; Scherer & Gustafsson, 2015). Moreover, writing is beneficial for female learners, who often perform better when verbal and reflective tools are integrated into

STEM education (Reilly, 2007). Through writing, students clarify misconceptions and develop personal ownership of learning, which can be particularly transformative in abstract subjects like mathematics.

Research has shown that incorporating writing into mathematics instruction benefits students, especially those who struggle academically (Braun, 2014 ; Cross, 2009; Madison, 2012; O'Connell et al., 2005). For instance, studies involving seventh-grade students have demonstrated that journal writing allows learners to explain their reasoning, explore multiple problem-solving strategies, and communicate their understanding in a non-threatening environment (Bruun et al., 2015). This approach not only enhances students' strategic competence but also provides teachers with valuable insights into their students' thought processes. Writing in mathematics encourages students to delve deeper into problem-solving and fosters a sense of ownership over their learning (O'Kelly, 2013). According to Braun (2014), by articulating their reasoning, students can develop a more profound understanding of mathematical concepts and see the subject as an evolving field where they can contribute new ideas. This process helps students reassess their knowledge, build upon existing understanding, and become more invested in their mathematical journey (Bangert-Drowns et al., 2011).

Moreover, Baxter (2008) notes that writing offers opportunities for both students and teachers to pause and reflect on their thoughts. It aids in organizing knowledge, making sense of problems, and encouraging deeper thinking (Braun, 2014; Cross, 2009). Through writing, students can communicate their ideas clearly, consolidate their thinking, and synthesize information to form a coherent understanding of mathematical concepts (Peterson, 2007; Pugalee, 2004). This practice not only improves their mathematical habits but also enhances their ability to see the underlying principles and reasoning in mathematics (Braun, 2014 ;O'Connell et al., 2005). The writing to learn Mathematics consider things that are not typically measured like metacognitive behaviours Pugalee, (2001), problem solving strategies Pugalee, (2004), and the long-term effects of writing in the classroom and how it might affect college and career readiness (Madison, 2012). Incorporating writing into mathematics instruction also supports vocabulary development (Bruun et al., 2015). By writing about mathematical terms, students can internalize definitions, use new vocabulary appropriately, and strengthen their overall proficiency in the subject (Dündar, 2016; Stonewater, 2002). Effective use of mathematical language enables students to communicate their thinking more precisely and engage more fully with the mathematical community (McCormick, 2010).

According to Kenney, Shoffner and Norris (2017) and Knox (2017) states, when students express their mathematical thinking through writing, it provides teachers with valuable information to identify misconceptions and tailor instruction accordingly. For example, asking students to explain why one fraction is larger than another can reveal their understanding of fraction concepts. This written communication fosters meaningful dialogue between students and teachers, enhancing the learning experience (Dündar, 2016). Teachers can utilize writing in mathematics as a diagnostic, formative, or summative assessment tool (Braun, 2014). Diagnostic assessments might involve students writing about their mathematical experiences, providing insights into their background and attitudes toward the subject. Formative assessments can include expository writing that promotes mathematical reasoning and helps teachers adjust instruction based on student needs (Braun, 2014; Santos and Semana, 2014).

Summative assessments might require students to summarize their understanding of a concept, demonstrating their mastery of the material (Knox, 2017). Pugalee (2004) states that, despite the recognized benefits, writing in mathematics classrooms has not been widely adopted. Porter and Masingila (2000) states that some educators express concerns about the additional time required to read and respond to student writing. However, research like that of Kenney et al.'s (2017), indicates that with proper training and support, teachers can effectively integrate writing into their instruction, leading to improved student outcomes. They reported that pre-service lecturers in particular, have shown increased appreciation for the value of writing in mathematics after engaging in targeted writing activities and reflections. In conclusion, writing to learn in mathematics not only enhances students' academic achievement but also positively influences their self-concept and interest in the subject. By embedding writing into mathematics instruction, educators can support students in developing a deeper understanding of mathematical concepts, fostering a lifelong appreciation for the discipline.

- *Writing as a Mode of Learning in Mathematics*

Incorporating writing into mathematics instruction transforms the learning experience from passive reception to active engagement. Through writing, students can explore mathematical ideas, clarify their understanding, and make connections between concepts. This process not only aids in retention but also encourages students to view mathematics as a dynamic and interconnected discipline. By expressing their reasoning in written form, students develop a more profound comprehension of mathematical principles and enhance their problem-solving abilities.

- *Impact on Academic Achievement*

The integration of writing into mathematics education has been linked to improved academic performance. By articulating their thought processes, students can identify gaps in their understanding and address misconceptions. This reflective practice enables learners to develop more robust problem-solving strategies and fosters a deeper appreciation for the subject. Moreover, writing assignments in mathematics encourage students to engage with the material more thoroughly, leading to increased motivation and academic success.

- *Influence on Self-Concept and Interest*

Engaging in writing activities within the mathematics classroom positively affects students' self-concept and interest in the subject. As students successfully communicate their mathematical reasoning, they build confidence in their abilities and develop a stronger sense of competence. This enhanced self-perception motivates learners to tackle more complex problems and persist in the face of challenges. Additionally, writing allows students to connect mathematical concepts to real-world applications, increasing their interest and appreciation for the subject.

- *Addressing Evaluation Anxiety*

Writing in mathematics serves as a valuable tool for reducing evaluation anxiety. By providing students with opportunities to express their understanding in a low-pressure format, educators can alleviate the stress associated with traditional assessments. This approach fosters a supportive learning environment where students feel comfortable exploring ideas and taking intellectual risks. As a result, learners are more likely to engage deeply with the material and develop a positive attitude toward mathematics.

- *Enhancing Vocabulary and Communication Skills*

Writing activities in mathematics classrooms contribute to the development of subject-specific vocabulary and communication skills. As students describe mathematical concepts and procedures in their own words, they internalize terminology and improve their ability to convey complex ideas clearly. This linguistic proficiency not only benefits their performance in mathematics but also enhances their overall academic communication skills. Furthermore, the ability to articulate mathematical reasoning is essential for success in collaborative problem-solving and interdisciplinary applications.

- *Implications for Teaching Practices*

The incorporation of writing into mathematics instruction necessitates a shift in teaching practices. Educators must design assignments that encourage critical thinking and provide meaningful opportunities for students to reflect on their learning. This includes creating prompts that require explanation of problem-solving strategies, justification of solutions, and exploration of mathematical concepts. Additionally, teachers should provide constructive feedback on students' written work to guide their development and reinforce understanding. By adopting these practices, educators can cultivate a classroom environment that values communication, fosters deep learning, and supports student achievement. In conclusion, Integrating writing into mathematics education offers numerous benefits, including enhanced academic achievement, improved self-concept, increased interest in the subject, reduced evaluation anxiety, and strengthened communication skills. By embracing writing as a tool for learning, educators can create a more engaging and effective mathematics classroom that supports the holistic development of students. This approach not only prepares learners for success in mathematics but also equips them with essential skills for lifelong learning and problem-solving.

- *Benefits of Writing to Learn Mathematics*

Integrating writing into mathematics education has emerged as a powerful pedagogical approach that enhances students' understanding, reduces anxiety, and fosters a deeper engagement with mathematical concepts. This strategy, known as "writing to learn," encourages students to articulate their thought processes, leading to improved comprehension and communication skills. This essay explores the multifaceted benefits of writing in mathematics, its applications in educational settings, and its relevance in the workplace.

- *Enhancing Conceptual Understanding*

Writing in mathematics compels students to organize their thoughts and articulate their reasoning, which deepens their conceptual understanding. When students explain mathematical concepts in their own words, they engage in reflective thinking that solidifies their grasp of the subject matter. This process not only aids in internalizing complex ideas but also reveals gaps in understanding, allowing for targeted instruction and support (Bicer, Perihan, & Lee, 2018).

- *Reducing Mathematics Anxiety*

Mathematics anxiety is a common barrier to student success. Writing activities, such as journaling, provide a private and non-threatening outlet for students to express their concerns and challenges. This reflective practice can alleviate anxiety by shifting focus from performance to process, enabling students to approach mathematics with increased confidence and reduced stress (Ramirez & Beilock, 2011).

- *Fostering Positive Attitudes and Engagement*

Engaging students in writing about mathematics cultivates a more positive attitude toward the subject. By encouraging personal expression and exploration, writing activities make mathematics more relatable and accessible. Students who may be hesitant to participate in verbal discussions often find their voice through written expression, leading to increased participation and a sense of ownership over their learning (Bicer, Perihan, & Lee, 2018).

- *Improving Communication Skills*

Effective communication is a critical skill in mathematics. Writing tasks require students to convey their reasoning clearly and logically, enhancing their ability to discuss and justify solutions. This practice not only benefits their mathematical proficiency but

also prepares them for collaborative problem-solving and interdisciplinary applications where clear communication is essential (Sumner, 2016).

- *Informing Instruction and Assessment*

Writing provides educators with valuable insights into students' thought processes, misconceptions, and learning progress. Analyzing written work allows teachers to tailor instruction to meet individual needs, address misunderstandings, and adjust teaching strategies accordingly. This formative assessment tool supports a more responsive and effective educational environment (Bicer, Perihan, & Lee, 2018).

- *Applications in Educational Settings*

Incorporating writing into mathematics instruction can take various forms, including journals, explanatory essays, and reflective prompts. These activities encourage students to connect mathematical concepts to real-life situations, fostering relevance and deeper understanding. For instance, students might write about how they applied mathematical reasoning to solve a practical problem, thereby reinforcing the applicability of mathematics beyond the classroom as confirmed by Graham, Kiuahara, & MacKay, (2020).

- *Relevance in the Workplace*

The ability to articulate mathematical reasoning is highly valued in the workplace. Employers seek individuals who can analyze data, interpret results, and communicate findings effectively. Writing skills in mathematics enable professionals to present complex information clearly, support decision-making processes, and contribute to collaborative projects across various industries (Van Dyke, Malloy and Stallings, 2015). Integrating writing into mathematics education offers a comprehensive approach that enhances learning, reduces anxiety, and prepares students for real-world applications. By fostering reflective thinking, improving communication skills, and providing insights for educators, writing serves as a powerful tool in cultivating mathematical proficiency and confidence. Embracing writing to learn in mathematics enriches the educational experience and equips students with essential skills for academic and professional success.

- *Applications of Writing to Learn Mathematics at the School Level and in the Workplace*

For nearly three decades, the use of writing across all levels of Mathematics instruction has been advocated in countries such as the United States by organizations like the NCTM and the Mathematical Association of America (Van Dyke, Malloy, & Stallings, 2015). Research by Beaver and Beaver (2011) and Van Dyke et al. (2015) indicates that few students entering university from secondary schools are adequately prepared or accustomed to writing in Mathematics, and many at the college level perceive writing in Mathematics classes as irrelevant. Beyond academic settings, the ability to write Mathematically holds practical value in professional environments and in fostering clearer public communication (Madison, 2012). Employers frequently list writing proficiency as a critical qualification for job performance, yet they express concern over the poor writing skills of numerous employees. In 2004, it was reported that inadequate writing abilities cost U.S. businesses more than three billion dollars annually in training expenses (Quible & Griffin, 2007). One example of using Mathematical writing to improve public discourse includes the written explanation of data-based visuals. Moreover, encouraging individuals to write using numerical data may empower them to critically examine and challenge misleading statistical claims (Madison, 2012).

It is essential that all learners develop the capacity for meaningful Mathematical communication (Baxter et al., 2005). Research by Baxter and colleagues on journal writing in seventh-grade Mathematics classrooms showed that writing can support struggling students by offering those who rarely contribute to verbal discussions an opportunity to take greater ownership of their learning (Baxter, 2008; Baxter et al., 2005). Often, lower-performing students are limited to routine exercises, but engaging them in Mathematical writing can nurture their understanding of concepts and enhance their problem-solving skills. Over time, incorporating writing into Mathematics instruction may assist these learners—who might otherwise disengage from the subject—in forming enduring Mathematical understandings (Baxter et al., 2005).

Similarly, Santos and Semana (2015) examined the role of expository writing in a cohort of eighth-grade students. Their study focused on how students interpreted, represented, and justified their ideas in formative assessment tasks. The findings revealed that, with continued practice, students improved their use of justifications and representations while reducing ambiguous responses. This suggests that combining expository writing with constructive feedback supports the growth of Mathematical reasoning skills. Van Dyke et al. (2015) also found that many university students are hesitant or even resistant to writing in Mathematics, often viewing it negatively. However, they propose that consistent writing practice in Mathematics during middle and high school may enhance students' perceptions and openness to such tasks in higher education. This is especially likely when early experiences with Mathematics writing are grounded in positive, conceptually rich assessments. Ultimately, both educators and learners stand to benefit significantly from embracing this instructional strategy (Van Dyke et al., 2015).

- *Effect of Writing to Learn Mathematics, Student's Meta-Cognition and Meta-Cognitive Connections (Affective Outcome)*

Meta-cognition is broadly understood as an individual's ability to reflect on and regulate their own thinking processes. It encompasses both awareness of one's cognitive strategies and the ability to control and direct them toward achieving learning goals. According to Paris and Paris (2001), meta-cognition involves the strategic management of one's own learning, including planning,

monitoring, and evaluating cognitive efforts. Within Mathematics education, meta-cognition plays a critical role by guiding how students process Mathematical tasks, solve problems, and revise their understanding when misconceptions arise (Cross, 2009; Knox, 2017). Studies have consistently shown a strong relationship between students' meta-cognitive awareness and their academic success in Mathematics. For example, Pugalee (2004) notes that students who utilize meta-cognitive strategies such as self-questioning, error analysis, and strategic planning tend to perform better in problem-solving activities. Cross (2009) and Knox (2017) support this view by asserting that writing to learn in Mathematics contributes significantly to a student's meta-cognitive development by promoting reflective thinking and enhancing conceptual clarity. Through this reflective writing process, students not only articulate their Mathematical reasoning but also identify gaps in understanding and restructure their cognitive frameworks accordingly. The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes that written communication in Mathematics classrooms offers a crucial platform for students to clarify their thinking and reflect on what they know or still find difficult. Writing in Mathematics serves as an instrument for enhancing cognitive and meta-cognitive connections. It enables learners to analyze their thoughts, evaluate their problem-solving methods, and refine their understanding, thereby supporting deeper engagement with content.

- *Meta-Cognitive Awareness and Knowledge*

Flavell's foundational work on meta-cognition, as cited in Palmer (2018), distinguishes between two major components: meta-cognitive knowledge and meta-cognitive experiences. Meta-cognitive knowledge pertains to the awareness of various elements of learning—knowledge about oneself as a learner (person variables), the characteristics of the task (task variables), and the strategies suitable for solving problems (strategy variables). These categories collectively influence how a student navigates academic challenges. Meta-cognitive experiences, on the other hand, are emotional and cognitive responses that arise during learning. These experiences may include frustration when a concept is not understood or a sense of achievement when a solution is discovered. Such reflections often prompt learners to reassess their strategies and improve their approaches to problem-solving (Pintrich, 2002). Writing tasks provide a platform for these meta-cognitive experiences, allowing students to articulate their internal cognitive states, evaluate their learning progress, and plan subsequent actions. Flavell suggests that deliberate practice is vital for enhancing meta-cognition, and one of the most effective forms of this practice is writing. Writing allows learners to observe and critique their thinking processes, which not only improves their ability to self-regulate but also nurtures empathy as they consider alternate viewpoints. As Palmer (2018) asserts, writing fosters both introspection and cognitive clarity, making it a vital tool in the development of meta-cognitive competence.

- *Self-Regulation and Strategic Learning*

Self-regulation is a core aspect of meta-cognition. Brown, as cited in Palmer (2018), highlights that learners who self-regulate are better equipped to adjust their learning strategies, manage cognitive load, and stay motivated. In Mathematics, this means being able to set goals, select appropriate problem-solving techniques, and evaluate the accuracy of their solutions. Pugalee (2001) points out that meta-cognitive beliefs, such as confidence in one's ability to solve problems, significantly influence student outcomes. Successful students are not only knowledgeable but also adept at controlling how that knowledge is applied in varied contexts. Schoenfeld's (2016) research expands on these ideas by introducing a triadic model of meta-cognition in Mathematics: knowledge of one's own thinking processes, regulatory control, and beliefs or intuitions. This model aligns with efforts to train students as independent thinkers who can monitor their problem-solving pathways, identify when they are off track, and persist in finding solutions. Writing, therefore, serves not just as a communication tool, but also as a scaffold that supports strategic decision-making in learning.

- *Meta-Cognition in Problem Solving and Learning Progress*

Lucangeli and Cornoldi, as discussed in Reilly (2007), differentiate between routine Mathematical procedures and complex problem-solving tasks. While the former may become automatic with practice, the latter requires high levels of cognitive flexibility and meta-cognitive involvement. Their findings suggest that students with stronger meta-cognitive skills are better at managing the demands of non-routine problems, making them more likely to succeed in Mathematics.

Moreover, research shows that students who reflect on their learning through writing tend to retain concepts longer and understand them more deeply. Cross (2009) notes that such practices could lead to transformative learning experiences, where students not only improve academically but also learn to make better decisions and comprehend complex ideas more effectively. Writing activities encourage students to structure their thoughts, assess their problem-solving strategies, and revise their mental models, thereby fostering lifelong learning habits.

- *Writing to Learn Mathematics and Meta-Cognitive Growth*

Writing to learn Mathematics is a strategic method that promotes meta-cognitive development by encouraging students to articulate their thought processes and reflect on their problem-solving approaches. Carr (2010) emphasizes that significant shifts in Mathematical understanding require students to dismantle and rebuild their conceptual frameworks. These changes can be catalyzed through reflective writing, which acts as a conduit for integrating new knowledge and resolving cognitive dissonance. Carr further argues that the complex nature of Mathematical learning calls for increased research into how reflective practices, such as writing, influence students' conceptual transformations. Writing to learn offers such an avenue by prompting learners to continuously evaluate and adapt their thinking, thereby reinforcing the metacognitive cycle of planning, monitoring, and evaluating. In summary,

integrating writing into Mathematics instruction offers a dual benefit: it enhances content comprehension and nurtures the meta-cognitive abilities necessary for lifelong learning. As students engage in writing-based reflections, they become more aware of their thinking patterns, more capable of regulating their cognitive strategies, and more confident in tackling complex Mathematical challenges. For educators, fostering these habits can lead to more engaged, autonomous, and successful learners.

- *Writing Strategies in the Mathematics Classroom*

In most Mathematics classrooms, instruction traditionally involves students practicing a series of exercises that reinforce the day's lesson. While such a structure helps solidify procedural fluency, it often does not engage students in deeper critical thinking or conceptual understanding. For instance, students may know how to compute the value of π as approximately 3.14, yet remain unaware of its derivation or significance. Similarly, they might apply the Pythagorean Theorem without understanding its geometric proof or fail to identify which measure of central tendency—mean, median, or mode—is most appropriate in varying contexts (Braun, 2014; Knox, 2017). To address this gap, educators are encouraged to go beyond conventional assessments like quizzes, homework, and tests by incorporating writing as a strategy in Mathematics classrooms. Through written tasks, students can provide detailed explanations, clarify their understanding, and refine their conceptual thinking. Writing exercises can take various forms, including expository reflections, analytical essays, personal narratives, or even creative expressions (Braun, 2014; Peterson, 2007). Such tasks not only deepen understanding but also enhance Mathematical communication. Examples of Mathematical writing include journal entries reflecting on daily lessons, step-by-step explanations of problem-solving processes, descriptions of abstract concepts in students' own words, open-ended reflections on learning progress, and even autobiographical accounts detailing their journey with Mathematics (Knox, 2017). Burns (2004) further suggests assigning open-ended questions and encouraging students to write summaries after completing a unit. These activities help solidify learning and can prompt metacognitive insights.

Additionally, creative approaches like writing Mathematical poems, especially question-and-answer formats that reflect on problems and their solutions, have been advocated by Peterson (2007). Burns (2004) and Wilcox and Monroe (2011) also propose collaborative writing activities, such as peer discussions prior to writing, shared vocabulary charts, and cooperative tasks like group-authored projects. These interactive strategies promote active engagement and peer accountability. When students share their written work with peers, whether in "writing circles" or paired feedback sessions, they learn to articulate their ideas more clearly. These exchanges not only build confidence but also reinforce Mathematical language and discourse (McCormick, 2010). As part of a formative assessment approach, lecturers can use a "think-write-share" technique, where students reflect individually on a prompt, write their responses, and then engage in discussion (Wilcox & Monroe, 2011). This fosters a collaborative learning environment that emphasizes both individual responsibility and group insight. According to Burton and Morgan (2000), the nature of Mathematical writing can vary depending on the purpose, audience, and disciplinary conventions. Similar to writing in other academic genres, Mathematical writing improves with practice and structured support. The National Council of Teachers of Mathematics (NCTM, 2000) underscores the importance of giving students opportunities to refine their Mathematical expression as part of their overall development in the discipline.

Stonewater (2014) emphasizes the need for clear instructional scaffolding to support Mathematical writing. For students who find it challenging to express Mathematical ideas, lecturers can provide checklists, modeling of effective and ineffective samples, and peer-assessment rubrics. These tools help students identify essential elements of strong Mathematical writing, including logical reasoning, clarity, and the accurate use of vocabulary. Moreover, lecturers can utilize performance-based writing tasks to assess learning in a more authentic context. Such tasks require students to apply Mathematical knowledge to real-world scenarios, fostering deeper connections between abstract concepts and practical applications (Kenney, Shoffner & Norris, 2014). Writing about problem-solving not only strengthens students' grasp of concepts but also enhances their ability to reason, explain, and reflect—skills that are vital for lifelong learning and real-world decision-making.

- *Mathematics Performance-Based Writing Assessment, Student Achievement, and Affective Outcomes*

Reforming assessment practices in education has become necessary at all levels, including standardized testing and classroom evaluations. Performance-based assessment (PBA) is considered a progressive approach as it evaluates both the learning processes and final outcomes of students. In line with contemporary educational ideologies, it aims to move away from traditional methods and emphasize real-world application, critical thinking, and student engagement through tasks like writing, behavioral observation, and product creation (Mislevy & Haertel, 2020). This type of assessment challenges students to tackle open-ended, complex problems independently and creatively (Alkharusi, 2017). Mathematics is particularly suited to performance-based writing assessments because it combines conceptual understanding, procedural fluency, and real-life problem solving. In Mathematics, PBA often involves inquiry, application of mathematical concepts, and hands-on tasks that foster deeper engagement (Wang, 2020). As McMillan (2018) explains, PBAs directly assess how students apply their knowledge to solve realistic tasks, which allows for authentic evaluation based on both task nature and context. Learners are expected to plan, construct, present original solutions, and justify their reasoning. According to Kunandar (2017), the goals of performance-based writing assessments include tracking student progress, identifying mastered and unmastered competencies, and providing actionable feedback for growth.

Effective PBAs are contextually grounded, linking classroom learning to real-world applications. They also emphasize students' ability to apply knowledge in practical settings, develop critical thinking, and generate meaningful outcomes (Alkharusi, 2017). Rule (2019) outlines key features of PBAs: they present real-world tasks, promote open-ended inquiry and meta-cognition,

involve collaborative learning, and give students agency in their learning process. Mueller (2021) emphasizes that performance-based writing assessments should reflect tasks that students might encounter in their everyday or future professional lives. These assessments allow learners to use acquired knowledge to navigate authentic challenges and articulate their understanding clearly and thoroughly. In the context of Mathematics education, writing-based performance assessments help evaluate student competence in explaining real-world problems, analyzing situations, and developing strategies. Project-based writing assessments are useful for examining student comprehension, application, and communication skills (Hamzah & Koni, 2019). These assessments require students to conduct in-depth investigations into real-life topics, enabling them to apply their mathematical reasoning in various formats. Mini-writing projects can be implemented as part of the assessment to measure student's procedural knowledge, conceptual understanding, and affective development. Such tasks aim to reveal how students relate emotionally to Mathematics and how motivated and confident they feel in their learning process.

Alternative-Solutions Worksheets (ASWs) are an innovative tool used in PBA. They are grounded in Polya's four-step problem-solving model, which includes understanding the problem, devising a plan, executing the plan, and reflecting on the result. The last step, "flashing back," is often neglected but is crucial for conceptual development and self-assessment (Jacobbe, 2020). Encouraging students to consider alternative solutions through writing enhances their reflective thinking and problem-solving agility (Lee, 2018). For example, Huntley and Davis (2016) found that high school students typically begin solving Algebra problems using symbolic manipulation, but with guidance, they incorporate other representations like graphs and tables. Students adept at using multiple representations rely less on their instructors to detect and correct errors. Similarly, Hwang et al. (2017) studied Taiwanese students using a digital whiteboard system and found those using diverse problem-solving strategies performed better. Herman (2019) examined the impact of teaching multiple representations on college students' Algebra skills. While students primarily relied on symbolic methods, they acknowledged the value of other representations for error checking. This finding underscores the need for structured teaching that encourages reflection and peer dialogue. Communities of practice—where students' diverse approaches are acknowledged and discussed—can be fostered through writing-centered performance tasks (Kwon, Park, & Park, 2017).

Characteristics of performance-based writing assessments include real-world relevance, higher-order thinking, concept application, collaborative learning, and inquiry-based exploration. These features align with current educational priorities that emphasize deeper learning and critical engagement. Students are encouraged to go beyond rote memorization and develop the capacity to transfer knowledge across contexts. As Andrade (2020) notes, PBAs foster a holistic view of student learning by assessing complete and meaningful performances. This approach is also beneficial for gifted education and curricula that emphasize analytical thinking (Moon et al., 2018). However, to ensure PBAs are effective and standardized, scoring tools such as rubrics are necessary. A rubric translates qualitative student performance into measurable indicators. As Demers (2019) and Perlman (2016) highlight, rubrics must clearly define evaluation criteria and be applied consistently to yield valid judgments. Rubrics can be analytic or holistic. Analytic rubrics evaluate specific components individually and combine the scores, whereas holistic rubrics assess the overall quality of the student's response (Moskal, 2018). Both types are useful for evaluating the complexity and completeness of student responses.

Instructional rubrics, in particular, help students understand expectations and guide them toward improvement. Andrade (2020) explains that rubrics support self-assessment and provide targeted feedback. Shepard (2021) confirms that using rubrics encourages reflective practice and fosters learner autonomy. Andrade's (2020) study demonstrated that student self-assessment using rubrics correlated with improved academic outcomes. Ultimately, performance-based writing assessments in Mathematics do more than evaluate student achievement; they nurture affective qualities like motivation, confidence, and persistence. When implemented thoughtfully, these assessments support meaningful learning and prepare students for real-world challenges by fostering critical thinking, communication, and self-regulation.

➤ *Effects of Triangulation Methods of Assessment on Student's Academic Achievement and Affective Outcome*

The concept of triangulation was first initiated in the social sciences field when Campbell and Fiske published a paper in 1952 that discussed the application of a multi-method matrix procedure to assess the validity of measures and traits in the psychological repertoire (Ghrayeb, Damodaran and Vohra, 2011). The process of triangulation involves multiple entities, assessing the same outcomes, using different methodologies to validate the findings. Triangulation is a process of combining methodologies to strengthen the reliability of a design approach. When applied to alternative assessment, triangulation refers to the collection and comparison of data or information from three different sources (forexample reciprocity with peer open-ended and project-based methods of assessment) in order to assess students academic achievement.

Creswell (2009) explains that a triangulation approach in research is one where the researcher bases knowledge claims on pragmatic foundations. In line with this view, the central aim becomes solving practical problems. From a pragmatic standpoint, both qualitative and quantitative strategies are integral to deriving meaningful insights or evidence about assessment-related issues. Therefore, the strength of this approach lies in its ability to blend the most effective elements of both paradigms to explore complex problems. Consequently, in the context of this study, triangulation is applied as a technique for evaluating and enhancing the validity of assessments regarding students' academic performance, self-concept, and interest in Mathematics.

Relying solely on one form of assessment—such as paper-and-pencil tests with closed-ended questions—tends to emphasize only one aspect of learning while ignoring others (Smith, Teemant, & Pinnegar, 2004). This narrow focus can result in misleading conclusions about a student's progress and overall learning outcomes. According to Wiggins and McTighe (2005), effective assessment should include a combination of strategies such as performance-based tasks (project-driven assessments), assessments that evaluate knowledge and skills (peer collaboration methods), and criterion-referenced tests (which may include open-ended formats). Triangulated assessment depends on diverse data sources and evaluation methods to confirm a finding, by showing that various independent measures converge or at least do not contradict each other. Despite its growing use, triangulation in assessment is still a relatively new methodology and therefore requires further empirical exploration and scholarly validation.

One of the key advantages of triangulation in the assessment process is its potential to enhance research credibility, reduce bias, and expose variations in outcomes to develop more substantiated claims. According to Mathison, triangulation can be implemented through four major strategies: (a) data triangulation, which includes variations in time, location, and participant perspectives; (b) investigator triangulation, involving multiple researchers; (c) theoretical triangulation, using different theoretical perspectives; and (d) methodological triangulation, which combines various methods of data collection (Ghrayeb et al., 2011). Specifically, data triangulation involves collecting evidence from multiple sources to examine the same phenomenon. When multiple sources yield similar findings, it enhances the validity of the outcomes. This process may require contributions from different individuals, locations, or timeframes. For example, Mathison noted that evaluating student learning outcomes at various times throughout the day and across different seasons can demonstrate whether consistent results support the validity of the learning outcomes.

Triangulation methods have gained significant popularity in social, behavioral, and related disciplines in recent times (De Vos et al., 2011). Additionally, they are now widely recognized as valid approaches in educational research (Ridenour & Newman, 2008). In this study, the choice of a mixed-method assessment strategy is motivated by the aim to triangulate both quantitative and qualitative findings. This allows for a more comprehensive exploration of the issues being investigated. Creswell (2009) emphasizes that pragmatic knowledge claims focus primarily on addressing the research problem. Thus, using both quantitative and qualitative techniques enables the researcher to gain a fuller understanding of the assessment issue. As De Vos et al. (2011) contend, triangulated assessment designs can eliminate various forms of bias, clarify the true nature of the phenomenon under investigation, and enhance the validity or quality of the findings.

Triangulated research design helps the researcher to attain a more nuanced understanding of a complex problem and allows for the examination of questions that might not be sufficiently answered through a single method (Lopez-Fernandez & Molina-Azorin, 2011). This assessment strategy adopts a problem-focused approach to more effectively investigate the issue at hand. Scholars who advocate for triangulated assessments argue that the combination of multiple methods offers a more complete and accurate representation of the phenomenon than using either approach in isolation (Creswell, 2012; De Vos et al., 2011). In the present study, the researcher concurrently collects data through both peer interaction and project-based assessments. These data sets are then merged to interpret the overall findings. The rationale for using a concurrent mixed-method design lies in its capacity to offset the weaknesses of one method with the strengths of another, thereby producing a more comprehensive understanding of the assessment problem (Creswell, 2012).

Additionally, triangulating data enhances the reliability and credibility of the study's findings. Incorporating alternative assessment strategies like triangulated assessments in Mathematics education provides all students with multiple entry points to learning. This allows diverse learners to showcase their understanding in various ways, leading to more equitable outcomes and improved student engagement. When learners are assessed using multiple formats, they are challenged to engage with rigorous content rooted in theories such as Constructivism and Cognitivism. This approach aligns with reformed curriculum visions and introduces innovative classroom assessment methods.

The types of challenging content involved in triangulated assessment include the understanding that intellectual skills are shaped within cultural and social environments, and that learning occurs within interactive, contextual settings. Learners bring prior knowledge and cultural backgrounds that influence their learning processes. Cognitive growth is also linked to self-awareness and metacognitive strategies. Deep comprehension depends on meaningful connections and promotes knowledge transfer across contexts. Overall cognitive performance is influenced not only by intellectual capacity but also by learners' dispositions and sense of identity.

In a review conducted by Muis (2004) on students' beliefs about Mathematics, evidence suggests that learners across all levels—including those at the undergraduate level—often hold counterproductive beliefs. These include the view that Mathematics is only about finding the correct answer and that Mathematical knowledge is transmitted passively by an authority figure such as a teacher or textbook author. Students also tend to believe that Mathematics is not rooted in logic or reasoning and that proficiency in the subject is an innate trait possessed only by a select few. Furthermore, many students feel incapable of independently constructing Mathematical knowledge or solving problems (Muis, 2004). Changing these beliefs requires instructional strategies that directly challenge and replace them. Four conditions are necessary for conceptual change to occur: dissatisfaction with existing beliefs, as

well as clarity, plausibility, and usefulness of the new beliefs. Raising awareness of one's own beliefs is also a vital step in this transformation. Moreover, the type of instruction students receive often mirrors and reinforces their beliefs (Muis, 2004).

Instructional and assessment models that reflect a constructivist philosophy can help shift students' perspectives toward more constructive beliefs about Mathematics. According to Muis (2004), constructivist-based instruction places Mathematical content in real-world and meaningful settings, emphasizes collaboration and group learning, encourages active knowledge construction, focuses on learning processes, and allows sufficient time for conceptual development. These instructional approaches foster the belief that Mathematics is a method of reasoning, that Mathematical knowledge is interconnected and relevant to other academic and practical domains, and that it can be developed over time with effort and perseverance. Additionally, they promote the idea that Mathematical understanding is not an inborn talent but a skill that can be constructed by the learner.

Recognizing that student beliefs influence learning outcomes, the NCTM standards recommend that, alongside evaluating students' general Mathematical knowledge, their beliefs about the subject should also be assessed. Research indicates that students' epistemological beliefs are connected to their learning strategies and overall achievement. As such, this study adopts an assessment approach designed to foster belief change in the context of learning Mathematics. By participating in varied assessment methods, students are provided with opportunities to articulate their beliefs about learning Mathematics and reflect on how those beliefs may have evolved throughout the course of the intervention.

Reciprocity is an interesting concept, not just for the field of communications only but in the mathematics, sciences and our life itself. Aside from communication study, reciprocity can be observed in Newton's third law of motion which states that, "to every action there is an equal and opposite reaction". When a billiard ball is hit, the ball will travel in a straight line and upon hitting another ball, it moves due to the force exerted on it by the first ball. When a water droplet drops into a pool of water, the impact will cause ripple to be spread outwards due to the gravitational force of the falling water droplet. Besides science we can also see the effect of reciprocity in our daily lives transactions. When one wave, smile or nod at someone, they often than not respond with the same or at times different positive gestures out of goodwill. While this may sound philosophical, it seems true that almost everything in life has a reciprocal nature in them including educational assessment.

Reciprocal Teaching was initially introduced by Palincsar in her doctoral research in 1982 (Reilly, Parsons, & Bortolot, 2014). The method has since been refined and widely applied, particularly within literacy instruction contexts (Department of Education and Early Childhood Development [DEECD], 2007, 2008). This teaching strategy was specifically designed to enhance reading comprehension by fostering collaborative learning among students as they engage with various texts to develop meaning and deepen understanding. As originally conceptualized by Palincsar and Brown, Reciprocal Teaching involves four core steps: predicting, clarifying, questioning, and summarizing (DEECD, 2008).

The predicting phase involves learners anticipating the forthcoming content of a text. These predictions are guided by their prior knowledge, textual structure, headings, visuals, and contextual clues. This stage helps maintain students' engagement, as they are often curious to verify whether their predictions are accurate. By predicting, students are encouraged to think proactively (DEECD, 2007, 2008).

During the clarifying phase, learners are prompted to recognize sections that they find confusing—these may include unfamiliar words, complex structures, or difficult concepts. Such challenges can cause students to lose the overall meaning of the passage. At this stage, students attempt to resolve these comprehension issues before rereading the text to regain understanding. The clarification step is particularly beneficial for students who have persistent comprehension difficulties, as it enables them to repair gaps in meaning and restore logical flow.

The questioning phase offers students the chance to delve into the core meanings of the text. Learners are encouraged to pinpoint key ideas and formulate questions based on these insights. Successfully generating relevant questions requires students to identify significant information within the material, making them more actively involved in the learning process. Instead of merely responding to the teacher's inquiries, students take initiative in creating and answering their own. This practice fosters self-monitoring of comprehension and strengthens their summarizing abilities.

In the summarizing stage, students synthesize and condense essential ideas from the text. Summarizing may take place across a sentence, paragraph, or entire passage, and it supports students in integrating and articulating critical information (DEECD, 2007, 2008).

Many students face challenges in solving Mathematical problems, particularly in identifying the appropriate operations and understanding the steps involved. Since Mathematical literacy significantly impacts students' performance, improving their comprehension of written Mathematical problems becomes a logical and necessary step for better learning outcomes (Marzano & Pickering, 2005; Booker, Bond, Sparrow, & Swan, 2004; Ludwig, 2000).

Reciprocal Teaching is a structured instructional strategy that has been demonstrated to improve comprehension (DEECD, 2007, 2008). It also enhances students' ability to manage complex tasks, increasing their confidence and motivation to read and learn (Department of Education and Training [DET], 2006). As a dialogic, evidence-based approach, Reciprocal Teaching has proven effective in advancing reading comprehension skills and supporting learners in tackling Mathematical word problems (Collen, 2011; Wessman Huber, 2011; Quirk, 2010).

Alvermann, as cited in the work of Palincsar and Brown and referenced by Reilly, Parsons, and Bortolot (2014), argues that an adolescent's perception of their academic competence directly influences their motivation to learn subjects like Mathematics. Furthermore, Alvermann contends that engaging learners in small group discussions and treating texts as tools for discovery rather than simply as sources of information is more effective in promoting meaningful learning experiences.

Informed by these insights, the current study adapts the principles of Reciprocal Teaching into a specialized peer assessment strategy called the Reciprocity with Peer Assessment Method. While this approach draws from the model presented by Reilly, Parsons, and Bortolot, it incorporates significant modifications to suit the educational context of the study. The adapted method features five distinct phases: metacognitive questioning, visualizing, solving, reflecting, and connecting.

This model is developed based on three interdependent components which are, metacognitive questioning, cooperative learning and systematic provision of feedback for corrective enrichment of the learners. Metacognitive questioning is seen as the vital component and is further divided into three types of questions, these are comprehension, connection and systematic questions.

The comprehension question, requires the learner to reflect on the type of mathematical questions they are being asked before solving it, what type of mathematical operations they may be required to use without concentrating on the details and what their answer might look like. Once again there is a heavy emphasis on using prior knowledge, the structure of the text, headings, content and illustrations or diagrams. The connecting question requires the learners to focus on this task and think about the similarities and differences relating to task that they have completed previously. This will help them differentiate between surface and deep mathematical structures of the problem. The systematic question prompts the students to think about how to solve the problem as well as the mathematical strategies to use in solving them.

During the visualization stage, the learner is required to list three groups of information. The first list contains words they are unfamiliar with, the second states all the facts they know, i.e. generally statements or values from the mathematical problem. The last list requires a higher order of mathematical thinking and asks the students to compile a list of the information they are yet to determine in order to successfully solve the problem. As part of this stage, learners are encouraged to work as part of a group. Group work provides an opportunity for students to talk and socially interact with their peers; it helps them to construct meaning and promotes learning and literacy. Once the learners have visualized all areas of deficit, they are encouraged to re-read the text to restore meaning.

At the solving stage learners actually solve the problem. Learners are provided with a number of problem solving options, though not directed to a specific problem solving strategy. This empowers the student to develop a solution which is pertinent to them as learners. The learners are required to represent their working and solution using pictures or diagrams, numbers and words. The reflecting stage is completed by the individual as a self-reflection. This stage requires learners to evaluate how they contributed to the group task. The learner is also required to reflect on the strategies they have selected and to evaluate how they would refine the process if presented with a similar problem. The learner is also asked to justify their answer. To further enhance the mathematical understanding of all the students in the class, at the conclusion of each lesson they discuss and reflect on the mathematical solutions that have been offered by each group.

The final component of the reciprocity with peer assessment method is connecting. Throughout the entire process learners are required to keep a written record of what they have completed under each of the four headings. This is the main area where the model deviates from the Reciprocal teaching model used in literacy. The connecting integrates reading writing and continuously reinforcing the importance of linking the learnt or assessed concept to real life problem situation. connecting is also thought to lead to improved comprehension, understanding and retention of subject area content and it provides an opportunity for corrective feedback which is necessary to help students develop (Siemon and Virgona, 2007).

Reciprocity in the form of working together with peers, and working together was sometime mediated by the lecturers in the class during lectures. Throughout this study, students are regularly instructed by the lecturers to talk to each other in structured ways (think-pair-share, turn-and-talk), to look back at the problem, explain their mathematical thinking, and to discuss their work or their mathematical thinking with other students. Students reflected on reciprocity with peers in three major ways: 'looking at work,' 'talking,' and 'listening.' Overall, these social interactions (reciprocity with peers) around problem solving and mathematical thinking, which is the cooperative component described in the written reflections, will be beneficial (helpful) to the students.

The study also wants to draw attention to the directionality of the peer interactions listed above. There are three main ways that the interactions occurred: from one peer to another peer or peers (i.e., 'talking to' or 'looking at'), to one peer from another

peer or peers (i.e., 'getting help from'), and as a bi-directional interaction to/from one to/from another peer or peers (i.e., 'sharing answers with peers' and 'discussing with peers'). 'Discussing solving the problem with peers then revising one's own approach' and 'deciding if one's own way or a peer's way is better' are examples that show students taking action ('revising one's own approach') or comparing methods (ways) that resulted from interactions (i.e., talking to, listening to, seeing the work of) with peers. Reciprocity with peers, provides evidence for the claim that students interacted with a peer or peers while problem solving and thinking mathematically will increase achievement, interest, motivation and value while problem solving in mathematics (Mayer, 2010).

Project-based method requires examinee to complete the project at their own time and submit at a specified date. Dunn and Price cited in Gbore (2013) has shown that project-based method (Home work/assignment) that permit students to complete assignments under preferred conditions of noise, light, design, mobility and time of the day improved student's achievement, attitude and conduct. Alonge (2004) asserted that this kind of technique of evaluation reduces stress and rote learning.

An examination of the submissions of Alonge (2004) on project-based technique of evaluation reveals that the technique tend to move away from the attitude of rote learning which aims at merely preparing students for examinations but propels students understanding of the concepts of the school subjects which were taught to them. The application of triangulation techniques as a complementary technique to closed-ended test technique of evaluating learning in Mathematics could engendered greater academic achievement on the part of the students in the classroom as it could reduce tension, anxiety, fear of failure and also motivate the students to maintain a good reading culture and increases the hope, interest and Self-concept of studying to passing examination without necessarily cheating.

It is therefore necessary that examiners should complement closed-ended test technique of assessing learning outcome with triangulation technique to reduce cheating behaviour, anxiety and fear of failing examination among the students. This will encourage them to add value to whatever they are learning and will increase their interest to learn Mathematics. The project-based technique should be used intermittently to keep the students busy at home towards making them to acquire mastery of the subject matter and to motivate them to learn. It is hoped that the implementation of these methods would lead to improvement in the reading culture of the students and subsequently great achievement in the learning of Mathematics and successful academic achievement as well as affective outcome.

➤ *Effect of Gender on Student's Academic Achievement and Affective Outcome*

Nevertheless, for the study of science base courses including Mathematics, gender disparity is a factor considered by scholars to affect student's performance in school. Empirical studies show that gender is a strong predictor of Mathematics achievement and that many differences have been reported between the achievements of boys and girls. Ngeng (2016) has observed that there are various opinions from psychologists about the concept of gender and its influence on academic achievement and performance. Thus gender issue has continued to be a relevant predicting variable in the academic achievement of learners at all levels. In specific terms girls are said to have advantage in a variety of verbal task than boys while boys are more successful with manipulative skills like solving mathematical problems. This is because to a greater extent, girls acquire language with greater speed and proficiency than boys and gender-related difference in verbal abilities appears very early as children began to talk.

Gender according to Komolafe (2011), has been interpreted to mean different concept in different contexts. For instance, gender may refer to certain biological characteristics as determined biologically, for instance, differences between a female and male. Gender has also been described not only as being physical and biological difference between men and women or male or female but also their roles arising thereof. Thus gender refers to the expectations people have from someone because they are either male or female and this is why this research work examines gender difference as one of the factors affecting student's performance in mathematics. In view of this argument among the scholars that gender is a determinant factor in student's achievement in Mathematics makes it imperative to be chosen as moderator variable or factor for this study vis-à-vis student's academic achievement and affective outcome in Mathematics.

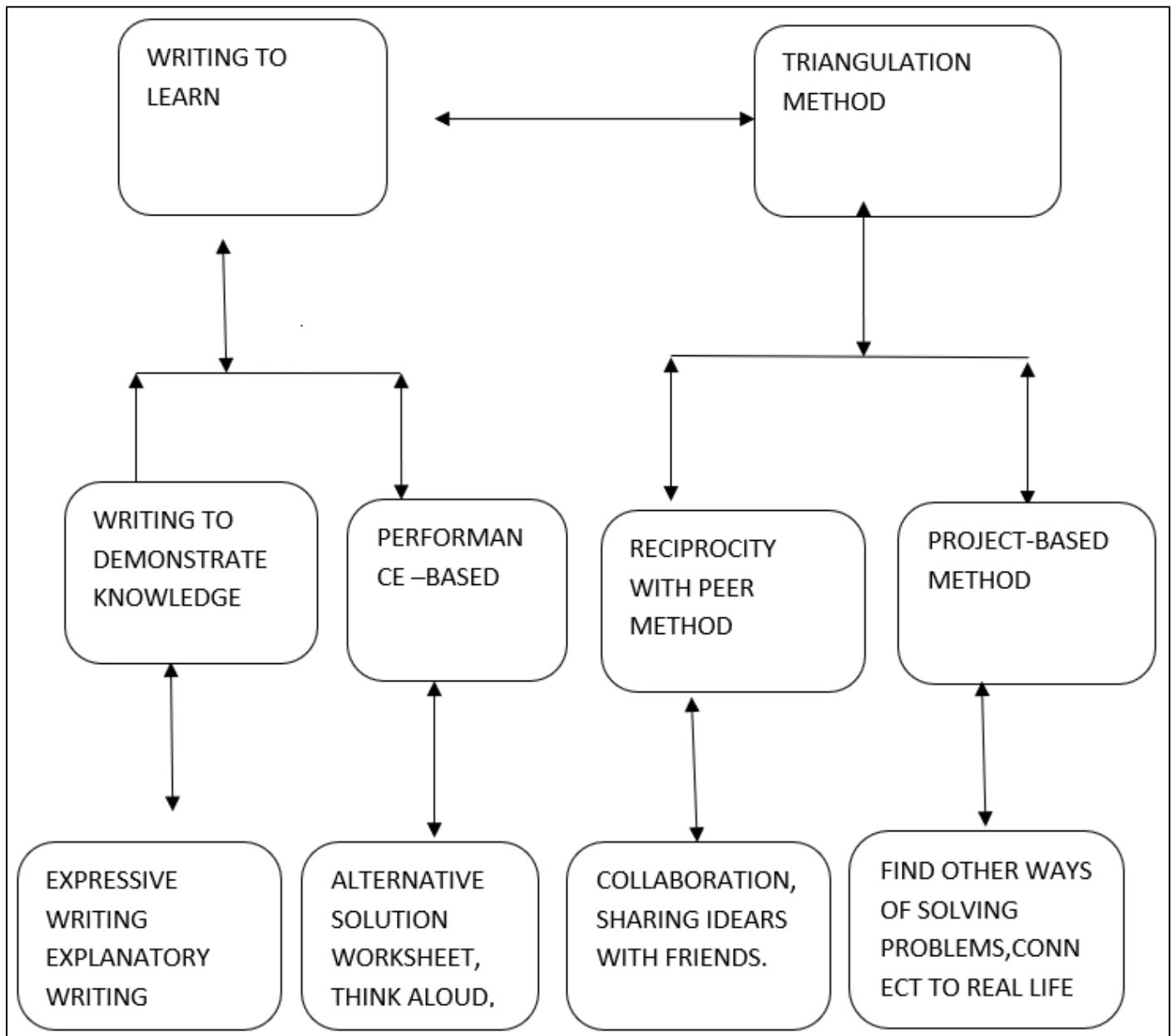


Fig 1 Writing to Learn and Triangulation Assessment Model

B. Theoretical Framework

The theories that gave backings to the study is reviewed. They are theory Of Andragogy by Malcom Knowles (1977) and Constructivism by Lev. Vygotsky (1978).

➤ Theory of Andragogy:

Malcolm Knowles' Perspective Historically, education was rooted in philosophical discourse led by thinkers like Confucius, Plato, and Aristotle, where the emphasis was placed on inquiry and dialogue rather than direct instruction. However, according to Knowles (1977), by the 7th century, this educational paradigm shifted toward structured learning with emphasis on reading and writing skills. As education evolved, particularly by the 12th century, it gradually became distinct from religious instruction, focusing on student-based teaching and progressing into the pedagogical framework that continues to dominate contemporary education systems. Pedagogy remained unchallenged until the post-World War I period when researchers began to recognize that adult learners required a different approach than children. This awareness led to the development of "andragogy," which Knowles (1977) defined as the "art and science of helping adults learn." Taylor and Kroth (2009) further highlight this theory's emergence as a foundational framework for adult education. Eduard Lindeman, a prominent figure in adult education during the 19th century, advocated for education to be a social rather than institutional process. He argued for the importance of informal education alongside traditional models—thus emphasizing learner-directed over teacher-dominated structures (Brookfield, 2015). His ideas helped lay the groundwork for what Knowles would later consolidate into a coherent theoretical model.

Knowles (1980) proposed six central assumptions underpinning adult learning: self-concept, prior experience, readiness to learn, orientation to learning, motivation to learn, and the need to know. These elements differentiate adult learners from children and form the cornerstone of andragogical instructional design (Merriam & Bierema, 2014). Self-directedness, the first principle, emphasizes an adult's natural preference for autonomy over dependency in the learning process. Conaway (2019) supports this view, noting that as individuals mature, they take increased responsibility for their learning. Unlike pedagogical learners, adult learners require learning environments that support and foster their independence while still providing structural support when necessary. The second assumption—learning from experience—posits that adults bring a wealth of knowledge from life and work which enriches the learning process. Knowles (1984) stressed that reflective learning enables adults to internalize concepts more deeply. Therefore, educational designs must actively integrate learners' prior experiences to construct new knowledge. Readiness to learn and orientation to learning are also central. Adults are often motivated to learn when they perceive a direct relevance of the content to their immediate personal or professional lives (Illeris, 2018). Unlike pedagogical methods which prepare learners for future application, andragogy encourages learning that directly addresses current, real-world problems.

The final assumptions—*intrinsic motivation* and the *need to know*—suggest that internal drivers such as self-fulfillment and career development are more powerful motivators for adult learners than external rewards. As Taylor and Kroth (2009) note, adults are more inclined to engage when they understand the value and purpose of what they are learning. Despite its strengths, andragogy has its critiques. Taylor and Kroth (2009) caution against assuming that all adults are self-directed or that all children are inherently dependent learners. The wide spectrum of ages considered “adult” also presents challenges for universal application. Furthermore, Conaway (2019) argues that adult learning characteristics can vary significantly, and instructional approaches must be flexible to accommodate these differences. A critical application of andragogy lies in the promotion of metacognitive strategies in learners. According to Livingston (2003), metacognition includes both self-appraisal (awareness of one's own learning) and self-management (ability to regulate one's learning activities). These are essential for learners to internalize new information and adapt their thinking patterns accordingly. Flavell's metacognitive framework—dividing it into person variables, task variables, and strategy variables—provides a useful lens for designing adult learning experiences. Person variables relate to understanding one's learning habits, task variables concern understanding the complexity of the task, and strategy variables involve knowing when and how to use specific learning methods (Palmer, 2018).

Through this framework, learners are encouraged to reflect critically on their learning processes, make informed decisions, and apply knowledge effectively. Holden and Yore (1996) affirm that this integration of metacognition enhances learning efficiency and academic performance. When aligned with andragogical principles, this fosters conceptual development, critical reasoning, and lifelong learning skills. Argumentation techniques, reflective writing, and collaborative learning strategies serve as effective methods for developing metacognitive awareness in adult learners. These methods not only promote deeper learning but also prepare learners for complex problem-solving in real-life situations (Livingston, 2003; Carr, 2010). The Andragogy Theory is related to this study of writing to learn Mathematics and Student academic achievement and affective outcome in Colleges of education. Using this theory to analyze the activity involved in the learners' engagement with writing to learn assessment, the following elements can be mapped.

This theoretical model effectively explains how various elements influence human activity, particularly in the context of teaching and learning. Human actions are often shaped by the tools and artefacts used in the activity—for instance, portfolios and performance-based writing serve as mediating tools in educational contexts. Engaging in these forms of writing facilitates meaningful activity that leads to specific educational outcomes. Within the context of writing-to-learn assessment strategies, five key process-related functions have been identified in Mathematics instruction. These include: supporting the development of conceptual understanding, enabling exploration of Mathematical ideas, integrating multiple representations and solution strategies, fostering self-directed learning, and encouraging the acquisition of problem-solving skills.

These functions became evident during the intervention phase of this study. The overall activity was also influenced by the educational community involved—including teachers, students, and departmental heads. Teacher and peer support played a crucial role in guiding students from a point of limited understanding to one of greater clarity and insight. Rules established by instructors also shaped how students engaged with the activity. For example, students were required to complete worksheets using digital applets and record their observations accordingly. The curriculum imposed constraints on the topics and scope covered during class, while the applets themselves also set certain parameters. Rubrics used for scoring defined performance expectations and could be adjusted to expand the evaluation criteria.

Students collaborated with both peers and lecturers in working toward shared learning goals. This collaboration reflects a division of labour within the learning environment. The theoretical foundation of this study draws upon andragogy, which encompasses both historical and context-specific dimensions of adult education. This theory helps explain the motivations behind context-dependent actions taken by individuals within an activity system. The central analytical unit in andragogical theory is the activity system—a group of people collectively engaging with a shared goal or object over time, using a variety of tools to achieve that objective.

An activity system, according to this framework, can be defined as “a sustained, goal-oriented, historically shaped, and tool-mediated human interaction.” This concept is applicable to the present study in the following ways:

- *Ongoing—*

The analysis of activity systems involves understanding how they operate across time. For example, Mathematics education is a long-standing activity system that has evolved historically and continues into the future. It is possible to trace the trajectory of learning in Mathematics and predict potential future developments.

- *Object-Directed—*

The activities under investigation in andragogical theory are always directed toward a defined purpose. In this study, the goal of the school system is learning, which is pursued through instructional practices and research—particularly through writing-to-learn strategies.

- *Historically Conditioned—*

All activity systems are influenced by the historical context of their development. The current assessment practices in schools are shaped by both institutional history and broader educational traditions. This study focuses on the integration of writing-based instruction and assessment, which has its roots in the historical evolution of pedagogical methods.

- *Dialectically Structured—*

The notion of dialectics describes interdependent relationships among the components of a system. When one component shifts, others are also affected, sometimes in unanticipated ways. For example, as students began to use writing as a central component of assessment, the roles and practices of lecturers, researchers, and students adapted in response to this shift.

- *Tool-Mediated—*

Human activity is invariably supported by tools, whether physical (e.g., software, lab equipment) or symbolic (e.g., language, Mathematics). In higher education, tools like textbooks, digital resources, and syllabi are used to facilitate learning. Writing assessment tools serve as mediators in the instructional process, influencing how educators assess student learning and how they conceptualize the learning activity itself. The use of triangulation assessment methods in this study helped structure classroom assessment in a way that affected both student engagement and instructional decision-making.

Human interaction is central to this model. Rather than focusing solely on individual actions, the theory emphasizes how adults collaborate to achieve shared educational goals using tools—such as writing-to-learn strategies. In this study, various stakeholders including teachers, students, researchers, and administrative personnel interacted with each other and the assessment tools to facilitate student learning. Ultimately, Knowles’ andragogical framework promotes a learner-centered approach that, when integrated with metacognitive strategies, offers a robust methodology for adult education. This perspective moves instruction away from passive knowledge transfer toward active, engaged learning, empowering adult learners to take ownership of their educational experiences and adapt effectively to dynamic learning environments.

➤ *Social Constructivism: Lev Vygotsky (1978)*

Lev Vygotsky formally introduced the concept of social constructivism in 1978, though the foundations of this theory were laid earlier by educational theorists such as John Dewey and Jean Piaget. Dewey emphasized the importance of inquiry in education, while Piaget contributed key ideas like assimilation, accommodation, and the role of cognitive schemas in learning. Together, these foundational ideas shaped the early stages of constructivist theory by highlighting how individuals process and structure their learning.

Vygotsky extended these ideas by arguing that social interaction plays a critical role in cognitive development. He proposed that higher-order thinking emerges from interactions between individuals and their social contexts. According to social constructivist theory, learning is not an isolated act but a collaborative process, structured around two key developmental levels: what a learner can do independently and what they can achieve with guided assistance. This distinction led Vygotsky to formulate the concept of the Zone of Proximal Development (ZPD).

The ZPD refers to the potential level of understanding that learners can reach with appropriate support from a more knowledgeable peer or mentor. This process is often supported through scaffolding, a method of instructional support that breaks tasks into manageable steps tailored to the learner’s current performance level. The ZPD, within the context of this study, is exemplified in the transition from abstract knowledge—such as understanding vertical and horizontal asymptotes of a hyperbola—to more concrete comprehension facilitated through alternative assessment strategies like writing-to-learn Mathematics.

Learning, from a constructivist perspective, involves active engagement and personal interpretation built from prior experiences. This theory emphasizes the importance of deep learning, problem-solving, conceptual development, and the ability to recognize and utilize patterns of meaningful information. It carries significant instructional implications, particularly in designing

student-centered, knowledge-centered, assessment-focused, and collaborative learning environments (Yilmaz, 2011; Treagust, Duit, & Fraser, 2014).

Unlike behaviorist theories that focus on observable responses to stimuli, constructivist learning involves active engagement of the learner to build new knowledge based on prior understanding and experiences. Students incorporate new ideas into their existing knowledge frameworks, thereby enhancing the quality and depth of Mathematics learning. Constructivism views learners as active constructors of meaning rather than passive recipients of information (Pritchard & Woollard, 2017).

This paradigm shift has transformed instructional and assessment practices. Rather than focusing solely on rote memorization or content coverage, instruction grounded in constructivism prioritizes conceptual understanding. Likewise, assessment is viewed not as a terminal event but as a continuous, integral component of teaching that aligns with classroom objectives and real-life challenges. From this perspective, meaningful assessment examines the learner's conceptual network rather than isolated facts, enabling students to demonstrate a deep understanding and provide ongoing evidence of their thought processes to both instructors and themselves (Black & Wiliam, 2018; Andrade & Heritage, 2018). Formative assessment—assessment for learning—has emerged as a cornerstone of constructivist pedagogical practice. This approach shifts the role of teachers and students alike. Teachers facilitate learning by activating prior knowledge, engaging in rich dialogue, using open-ended tasks, peer/self-assessment, and concept mapping. These strategies support the transfer and application of concepts in real-life contexts. Moreover, sharing learning goals and success criteria with students and offering timely, constructive feedback are crucial in helping learners refine their understanding (Wiliam, 2017).

Unlike the traditional assessment paradigm, which focuses on factual recall and grading, constructivist assessment aims to enhance learning by involving students actively in the process. This aligns with the growing advocacy for assessment to be a participatory and developmental activity that contributes directly to learning. Consequently, constructivism has attracted significant attention for its implications for teaching and assessment practices in Mathematics education (Nieminen & Tuohilampi, 2020). This study adopts the cognitive constructivist theoretical lens, which focuses on internal mental processes rather than external stimulus-response patterns. According to Piaget's theory, learning is both an active and reflective process that involves assimilating new information into existing cognitive structures (schemas) and accommodating these structures to incorporate novel experiences. The learner, in this context, plays a central role as an agent in their own learning, using prior knowledge, deductive and inductive reasoning, self-monitoring, and self-regulation to build knowledge (Boekaerts & Corno, 2014). Assessment from this perspective serves as a means to understand what learners know, how they apply that knowledge, and whether they can transfer it to solve authentic, real-life problems. Assessment, therefore, is not a one-off event at the end of a lesson but a dynamic, ongoing process of gathering and interpreting evidence to enhance learning. Such a comprehensive approach is essential in Mathematics education, where higher-order thinking, procedural fluency, and real-world application are critical (Torrance, 2017). Cognitive constructivists advocate the use of alternative assessment methods—such as writing to learn Mathematics—that go beyond multiple-choice tests and capture more complex aspects of student learning.

Furthermore, this theoretical orientation underscores the significance of continuous feedback and self-assessment in fostering learning. Self-assessment, in particular, allows learners to construct and reconstruct their knowledge in light of feedback and evolving understanding. Good assessment practices, therefore, emphasize conceptual learning and the application of knowledge rather than rote memorization. Mathematics educators are encouraged to design assessments that prompt students to create new Mathematical ideas, conduct investigations, analyze and interpret data, report findings, and apply skills in meaningful ways (Dreher & Kuntze, 2015). When considering knowledge application in non-school settings, the social constructivist perspective becomes particularly relevant. Writing as a tool for learning in Mathematics provides a platform for students to refine and express their Mathematical understanding effectively. The goal of strengthening Mathematical communication aligns with policy priorities in Nigeria and globally. As outlined by the National Council of Teachers of Mathematics (NCTM), communication is one of the five essential process standards that support the development of Mathematical proficiency (NCTM, 2014). Writing enhances the learner's capacity to make connections between prior and new knowledge, thereby supporting internalization and retention of concepts. Writing activates multiple levels of cognitive engagement—motor, sensory, and cognitive—and creates conditions that foster metacognitive thinking. This form of thinking encourages learners to reflect on their reasoning processes, which in turn deepens Mathematical understanding. As such, writing is recognized as a unique and effective medium for supporting reasoning and fostering deeper learning in Mathematics classrooms (Klein, 2016; Bangert-Drowns et al., 2020).

C. Empirical Studies

This section presents a synthesis of existing research on the benefits of alternative assessment practices, the implementation of writing-to-learn strategies, and triangulation methods of assessment, particularly in relation to students' academic performance and affective outcomes in Mathematics at the tertiary education level.

➤ *Research on Alternative Assessment Practices, Academic Achievement, and Affective Outcomes in Mathematics*

Mathematics has long been recognized not only as a core academic subject but also as a critical thinking framework for problem-solving, scientific inquiry, and structured reasoning (Onwuachu & Nwakonobi, 2009). In contemporary times, there has been a paradigm shift in educational evaluation globally with growing emphasis on assessment as a tool to support learning rather

than merely a measure of learning. This has led to the development of assessment typologies such as “assessment for learning” (AfL), “assessment of learning” (AoL), and “assessment as learning” (AaL), each offering distinct contributions to educational development (OECD, 2015; Bennett, 2011). In Nigeria, however, empirical evidence has consistently shown that current assessment practices are largely summative, prioritizing end-point evaluation rather than formative feedback that supports continuous learning (Okon, 2020). While summative approaches dominate, research strongly supports the pedagogical power of formative and performance-based assessments in Mathematics and science education. For instance, a study by Dessie (2015) investigated the practice of AfL among science teachers in secondary schools within the East Gojjam Zone of Ethiopia. Employing a concurrent mixed-methods design, data were collected from 153 randomly selected science lecturers using questionnaires, interviews, and observations. Statistical techniques including ANOVA, MANOVA, and multiple regression were employed for analysis. The study revealed a minimal implementation of formative assessment due to a lack of training and technical expertise among teachers. The findings emphasized the need for sustained professional development to enhance lecturers' capacity for integrating AfL strategies into regular instructional practice, thereby promoting better student outcomes.

In a related investigation into student perspectives on assessment, Fernandes, Flores, and Lima (2012) conducted a qualitative case study in a Portuguese university to explore the impact of project-led education (PLE) on learning and evaluation. Their research focused on the reflections of engineering students engaged in a PLE framework. Using open-ended survey responses and focus group discussions, the researchers gathered data from 40 students over multiple academic sessions. Their results indicated that students found formative assessment more valuable than summative evaluation. Participants emphasized that continuous feedback facilitated deeper learning and allowed them to connect classroom knowledge with practical, real-life scenarios—thereby supporting the core principles of alternative assessments (Fernandes et al., 2012). These empirical findings align with recent research which has highlighted the transformative potential of alternative and formative assessments in Mathematics education. Studies have demonstrated that when students are assessed through project work, writing-based activities, or performance-based tasks, they engage more deeply with content, develop better conceptual understanding, and exhibit increased motivation (Chappuis & Stiggins, 2019; Black & Wiliam, 2018). In contrast to conventional testing methods, such practices promote analytical thinking, self-regulation, and personal accountability, which are crucial for academic success in Mathematics. Additionally, AfL practices offer opportunities to address affective outcomes such as motivation, self-efficacy, and interest in Mathematics. When students receive timely, constructive feedback, their academic confidence improves, which in turn positively affects their engagement and willingness to persist in problem-solving tasks (Ajayi & Olatunji, 2017; Irons, 2016). For example, the use of descriptive feedback instead of numerical grades has been linked to higher student autonomy and better learning gains (Sadler, 2014). In summary, the reviewed studies demonstrate that assessment strategies rooted in alternative, formative, and performance-based frameworks significantly impact both cognitive and affective learning outcomes. The transition from summative-heavy assessment culture to a more balanced system that incorporates AfL and AaL requires targeted policy interventions, consistent teacher training, and curriculum alignment to ensure successful integration into classroom practice.

➤ *The use of Writing-to-Learn Assessment Tools and their Effects on Academic Achievement and Affective Outcomes in Mathematics*

The integration of writing into mathematics instruction has garnered attention as a means to enhance students' understanding and engagement. Writing-to-learn strategies, such as mathematical journals and explanatory writing, encourage students to articulate their thought processes, leading to deeper comprehension and improved problem-solving skills. Bicer et al. (2018) conducted a meta-analysis examining the effects of writing interventions on students' mathematical attainment. The study revealed a moderate positive effect size (0.42), indicating that incorporating writing into mathematics instruction can significantly enhance students' achievement and attitudes toward the subject. In a study focusing on middle school students, Palma (2018) found that writing activities in mathematics classrooms allowed students to express their understanding more clearly, aiding teachers in identifying misconceptions and tailoring instruction accordingly. Similarly, Craig (2016) observed that when students wrote explanatory paragraphs in mathematics, they became more aware of gaps in their reasoning, prompting reflective thinking and correction of misunderstandings. The use of writing-to-learn strategies also positively influences students' attitudes toward mathematics. Engaging in writing tasks helps students develop a more positive disposition toward the subject, increasing their motivation and confidence. For instance, Hebert and Powell (2016) reported that fourth-grade students who received explicit instruction in writing mathematical explanations demonstrated improved attitudes and performance in mathematics. Sagaskie (2014) explored the impact of an Alternative Solution Worksheet (ASW) on ninth-grade students' problem-solving performance. The ASW encouraged students to generate multiple solutions and reflect on their problem-solving strategies. The study found that students using the ASW exhibited enhanced problem-solving abilities and demonstrated higher-order thinking skills, as outlined in Bloom's Revised Taxonomy.

Authentic assessments, such as mini-projects, have also been shown to improve students' learning outcomes in mathematics. Fauziah et al. (2018) implemented authentic assessments in statistics learning and observed that students actively participated in the learning process, both inside and outside the classroom. The use of mini-projects allowed students to connect mathematical concepts to real-life situations, enhancing their understanding and application of statistical knowledge. Performance assessment-driven instruction has been linked to improved attitudes and achievement in mathematics. Arshin (2015) investigated the effects of performance assessment-driven instruction on senior high school students in Ghana. The study revealed that students exposed to performance assessments demonstrated better problem-solving abilities and increased confidence in mathematics compared to those taught using conventional methods. Similarly, Avis (2014) conducted a study in the West Indies to examine the impact of

performance assessments on students' interest and achievement in science. The findings indicated that students who participated in performance assessments showed significant improvements in academic performance and developed more positive attitudes toward the subject. These studies collectively underscore the efficacy of writing-to-learn assessment tools in enhancing both academic achievement and affective outcomes in mathematics education. By fostering deeper understanding, promoting reflective thinking, and improving attitudes toward mathematics, writing-to-learn strategies serve as valuable components of effective mathematics instruction.

➤ *The use of Triangulation Methods of Assessment and the Effect on Academic Achievement and Affective Outcome of Students in Mathematics*

Triangulation, as a research and assessment strategy, involves the use of multiple methods or data sources to investigate a phenomenon, ensuring credibility and comprehensiveness of results. In educational assessment, triangulation commonly integrates at least three assessment approaches quantitative, qualitative, and observational—offering a fuller understanding of student learning and behaviour. Its application addresses limitations inherent in single-method evaluations, especially in disciplines like Mathematics where cognitive, metacognitive, and affective dimensions coexist and influence student performance. One of the notable studies highlighting the effectiveness of triangulation in Mathematics education was conducted by Reilly, Parsons, and Bortolot (2014). Their action research adapted the reciprocal teaching model, originally developed by Palincsar and Brown for literacy, into the Mathematics classroom. The study was conducted at Sunshine College in Melbourne, Australia, involving two Year Seven Mathematics classes controlled for variables such as ability, gender, and behaviour. One group applied the reciprocal teaching model (involving prediction, clarifying, solving, and summarising), while the other used conventional problem-solving strategies. Each group was presented with identical tasks, and their responses were evaluated using multiple methods including rubrics, observational data, and qualitative feedback a clear example of triangulated assessment. The results demonstrated that students in the reciprocal teaching group, although slower to respond, displayed greater problem comprehension, improved expression of reasoning, and stronger retention of conceptual knowledge. In contrast, the control group, though faster, performed with less accuracy and had difficulty articulating their methods. The study concluded that the triangulated reciprocal teaching model significantly enhanced students' mathematical literacy and reflective thinking, thus supporting both cognitive and affective development.

A research effort by Knight, Kotys-Schwartz, and Pawlas (2014) explored how engineering competencies are cultivated using triangulation as a strategic assessment method to measure student outcomes in a Capstone program at the University of Colorado. Data collection occurred at the conclusion of a team-oriented, industry-sponsored Capstone design course. The evaluation of both technical and professional abilities of engineering students was conducted by industry professionals and academic mentors, and these evaluations were cross-referenced with students' own self-assessments. The study analyzed the congruities and discrepancies among the different assessments and discussed their significance in refining and evaluating the Capstone curriculum. A total of 135 students, 9 mentors from the industry, and 10 faculty advisors engaged in the two-semester Mechanical Engineering Capstone design project. Student teams were formed and assigned industry-driven projects, with some being newly initiated and others continued from prior cohorts. Each team received ongoing guidance from both faculty and industry mentors through weekly sessions. Throughout the academic year, teams were expected to meet various benchmarks such as design evaluations, submission of technical documentation, and presentation at a final exposition. These benchmarks served as effective points for collecting diverse performance assessments. The initial phase of triangulation data analysis focused on identifying consistencies in evaluations provided by the three groups of raters.

The second phase involved examining whether qualitative, open-ended feedback aligned with the numerical scores obtained. In the third stage of the triangulation analysis, the focus shifted to identifying discrepancies among the ratings. A reliable way to analyze such differences is through statistical procedures, such as the one-way ANOVA. This method was applied to compare ratings of Teamwork skills, particularly to assess if the 4.12 rating from industry mentors was significantly lower than those given by faculty and student raters. The researchers concluded that using triangulated data from multiple evaluators can offer a valuable approach to Capstone program assessment, providing a more holistic view of how engineering skills are developing among students. The study also discussed how triangulated assessments could help enhance a junior-level course. Recommendations for future research included the importance of gathering sufficient data to allow statistical comparisons between rater groups, improving the coordination of assessment focus areas across different instruments, and conducting comparative studies across various Capstone programs.

A similar perspective was adopted in Gbore's (2013) study in Nigeria, which examined the relative effectiveness of three triangulated evaluation techniques closed book, open book, and open-time testing—on students' academic performance in Integrated Science (a core STEM subject). The study used a quasi-experimental, counterbalanced design, involving 300 secondary school students. Using data from continuous assessment records and standardized tests across a three-year period, Gbore found that while closed-book tests yielded the highest academic scores, the open-book method enhanced deeper conceptual engagement and reduced test anxiety. Importantly, combining both approaches was recommended as it promoted authentic assessment while minimizing examination phobia and academic dishonesty—two factors closely tied to affective outcomes.

In the context of higher education, Ghrayeb, Damodaran, and Vohra (2011) at Northern Illinois University demonstrated the practical use of triangulation in validating learning outcomes within an engineering program. Using a design project as a case study,

the researchers employed three raters, open-ended surveys, and quantitative performance metrics. Triangulated data were analysed using ANOVA and thematic coding. The convergence of multiple feedback mechanisms revealed consistent findings across raters, enhancing the reliability of outcome-based assessments. The study emphasized that triangulation is crucial not only for assessing academic mastery but also for identifying areas of instructional improvement and promoting learner confidence and motivation. Further supporting the value of triangulated assessment is Nelson's (2010) work at Austin College in Texas. This comprehensive study spanned four years and utilized a wide range of data sources, including national surveys (NSSE, YFCY, CIRP, CSS), institutional assessments, and qualitative faculty feedback. Nelson's approach involved cross-referencing national data with internal course evaluations and strategic planning reports, creating what he termed an "assessment helix." This iterative model allowed for continuous feedback loops among students, faculty, and administration, increasing faculty engagement and institutional transparency in assessment processes. Importantly, triangulation here contributed to measurable gains in critical thinking, oral communication, and quantitative reasoning among students. In addition to performance metrics, affective outcomes were also significantly influenced by triangulated assessment strategies. Nelson's study reported that students involved in multidimensional assessment environments developed a stronger sense of ownership, motivation, and value toward learning, aligning with constructivist and humanistic pedagogical frameworks. The findings align with Coats and Stevenson's (2006) framework of integrating staff development, curriculum advancement, and student engagement in a triangulated structure, making assessment a community-oriented and human-centred practice rather than a bureaucratic requirement.

Despite these promising findings, some researchers, including Thomas et al. (2005), caution that the validity of triangulated results still depends on the consistency of application and alignment of assessment tools with learning objectives. Furthermore, student preferences remain mixed; while many appreciate the feedback-rich nature of triangulated assessments, some students still lean toward traditional methods due to their familiarity and perceived objectivity toward traditional, summative grading. Therefore, while triangulated methods promote deeper engagement, metacognitive development, and holistic understanding, their effectiveness depends on strategic implementation, instructor training, and institutional support. In conclusion, triangulation in Mathematics assessment offers a balanced approach to evaluating learning by combining performance-based, reflective, and standardized tools. It enhances both academic achievement and affective outcomes, providing students with multiple pathways to demonstrate understanding while fostering a culture of feedback, reflection, and self-regulation. Its successful adoption requires intentional instructional design, assessment literacy among educators, and alignment with constructivist learning goals. As Mathematics instruction moves away from rote memorization to problem-solving and conceptual reasoning, triangulation remains a cornerstone for reliable, inclusive, and meaningful evaluation.

➤ *Effect of Gender on Student's Academic Achievement and Affective Outcome with Regards to the use of Alternative Assessment Methods*

Gender has long been regarded as a critical variable influencing academic achievement and affective outcomes in education, particularly in Mathematics. Several empirical studies have explored how male and female students differ in their learning preferences, response to instructional strategies, and academic outcomes when exposed to alternative assessment methods such as writing-to-learn, triangulation, and performance-based assessments. Reilly (2007) conducted a mixed-methods study involving 293 middle school students in Western Pennsylvania to investigate students' perceptions of writing in Mathematics. The findings indicated that students, especially those struggling academically (those with lower letter grades such as C, D, and F), viewed writing as an effective tool for enhancing their understanding. Importantly, the study revealed that female students were more favorably disposed toward writing-based Mathematics instruction. Writing seemed to align with their verbal learning styles, allowing them to reflect on concepts, articulate solutions, and build stronger cognitive links. Female students demonstrated deeper conceptual connections and a more positive affective disposition towards Mathematics learning through writing compared to their male counterparts. In a related analysis of quantitative outcomes, data presented in Table 4.21 of the referenced study showed that female students in the experimental group who were exposed to alternative assessment strategies (including writing-to-learn and triangulation) outperformed male students in both academic achievement and affective outcomes. The mean post-test score for females was 73.83 compared to 69.00 for males, and their affective outcome scores were also higher (72.73 vs. 71.64). This trend reversed in the control group where males slightly outperformed females, highlighting the positive impact of alternative assessment on female learners.

Additionally, qualitative findings supported the view that female students made more meaningful and reflective connections through their writing. Their responses demonstrated a deeper engagement with the mathematical content, suggesting that writing enhanced metacognitive awareness and personal relevance in learning. These connections were often linked to real-world scenarios, future professions, and personal learning goals—factors that reinforced affective commitment to the subject. On the other hand, male students in the same studies appeared to demonstrate a surface-level understanding and showed less engagement with the reflective process fostered by alternative assessments. The implication is that while both genders benefit from innovative assessment techniques, female students tend to derive greater academic and emotional gains particularly in tasks that require expression, articulation, and introspection.

Further supporting this perspective, Ngeng (2016) noted that gender remains a persistent predictor of performance in Mathematics. He emphasized that girls often outperform boys in verbal tasks, which aligns with the strengths required in writing-based assessment strategies. Conversely, boys have been found to excel more in manipulative and procedural tasks, which might

explain their comparative underperformance in writing-centered Mathematics instruction. Similarly, a study conducted using Two-Way MANOVA confirmed that while both writing-to-learn and triangulation assessment methods significantly impacted academic achievement and affective outcomes, there was a significant difference between genders favoring female students in the experimental group. Female students exhibited more substantial improvements in academic and affective domains, possibly due to better engagement with writing, feedback, and self-assessment practices encouraged by the alternative methods. Finally, the narrative analysis of students' reflective writings indicated that female students were more consistent in using metacognitive strategies such as "flashback," self-monitoring, and reflective feedback. These strategies enabled them to better internalize mathematical concepts and problem-solving approaches, further solidifying the effectiveness of alternative assessment in addressing gender disparities in Mathematics learning. In summary, the review of literature and empirical evidence clearly demonstrates that gender plays a significant role in determining the academic and affective outcomes of students when alternative assessment methods are applied. While all students benefit from diverse assessment strategies, female students in particular seem to thrive under conditions that promote reflective thinking, verbal expression, and self-directed learning. This underscores the importance of adopting gender-sensitive approaches to instructional and assessment practices in Mathematics education.

D. Summary of Literature Review

The reviewed literature encompasses conceptual and theoretical frameworks, as well as empirical studies on writing as a learning tool in mathematics and the application of triangulation methods in assessment within Nigerian tertiary institutions. The focus of reviewed studies within the conceptual framework, was on types of assessment, writing to learn Mathematics and student's academic achievement and affective outcome, the use of triangulation method of assessment was the benefits of using reciprocity with peer and project-based methods of assessing students achievement in Mathematical concepts, procedural understanding as well as affective outcome and gender effect. Various tools for data collection and data analysis were used in these studies. These studies highlight the effectiveness of alternative assessment methods, the role of writing in enhancing mathematical understanding, and the importance of triangulation in validating assessment outcomes. Additionally, gender differences in response to writing as a pedagogical tool are examined, emphasizing the need for inclusive instructional strategies.

Furthermore, triangulation-based assessment and writing as a form of instructional designs could help students improve their skills regarding using multiple representations and translating their understanding and how much they have been able to learn a particular concept. This in turn will help the lecturers to plan the lesson to improve upon students learning. The effectiveness of using writing to learn as both an assessment tools and instructional method in Mathematics classrooms and the use of triangulation method of assessment was demonstrated in many experimental studies. The review of empirical studies obviously revealed that, a lot of researches has been conducted on writing to learn as an assessment tool and utilizing triangulation as methods of assessment is not a strange concept to measurement and evaluation research. However, no studies as at the time of this study within the reach of the researcher have utilized writing to learn Mathematics as an assessment and instructional tool as well as using triangulation method of assessment to compare the effect on student's academic achievement and affective outcome within and outside Nigerian tertiary institutions. Examination bodies such as WAEC/NECO seems not to use triangulation methods in the assessment of student's academic achievement. Consequently, they use traditional method in assessing the students in the various schools across the country.

Most of the studies reviewed used only one form of writing while in this research, two forms which are writing to demonstrate knowledge and performance-based writing that cuts across assessment in the three domain of learning was used. This created situations in which true, comprehensive and complete assessment of student's academic achievement is done in tertiary institutions in Nigeria. Conclusively, a dangerous precedence and loop hole in the current National Policy on Education which seems not to have a firm grip on the principles and practice of assessment in the three domain of learning across the country has created the problem of educational standard. This in turn has made examination bodies (WAEC, NECO, NABTEB, etc) to be at a cross road as to what the real continuous assessment score of Nigerian students should be. However, the concept of using writing to learn as assessment tool and triangulation method of assessment which is for now a uniquely "strange" phenomenon in the Nigerian measurement and evaluation system might be a solution to problem of teaching problem solving and communicating problem solutions through writing as discussed in the preceding review of the literature which is the gap this study wants to fill.

Regardless of the problem-solving approach chosen, teaching problem solving and the subsequent sharing of ideas through writing will benefit both the lecturers and the students. The lecturers will gains insight into student's conceptual understanding of Mathematical concepts as well as any misconceptions held by students. By participating in problem-solving activities and writing about the results, students develop a deeper interest, self-concept, motivation, attitudinal change, value, meta-cognition and understanding of Mathematics. Therefore, the study on the effect of writing to learn Mathematics and triangulation evaluation approach on student's academic achievement and affective outcome in Colleges of Education, is necessary.

CHAPTER THREE METHODOLOGY

The detailed description of the method that was used in this study is presented in this chapter under the following headings: design of the Study, Area of the study, population of the study, sample and sampling technique, instruments for data collection, validation of instrument, reliability of the instrument, experimental procedure, method of data collection, method of data analysis and decision rule.

A. Design of the Study

The study employed quasi-experimental, pretest posttest non-randomised control group design. Quasi-experiment is an experimental design where randomization of subjects to experimental and control groups is not possible (Nworgu, 2006). Intact classes were therefore used for the study. The use of intact classes was necessary in order not to disrupt the normal class periods. The design is diagrammatically presented as shown below:

➤ Design of the Experiment

Group	Pre-test	Treatment	Post-test
Writing to learn	O_1	$X_a (X_1 - X_2)$	O_2
Triangulation Method	O_1	$X_a (X_3 - X_4)$	O_2
Traditional Method	O_1	X_b	O_2

Fig 2 Design of the Experiment

• Key:

- ✓ O_1 = Pretest
- ✓ O_2 = Post test
- ✓ X_a = Treatment administered to writing to learn and triangulation method group
- ✓ X_b = Treatment administered to traditional method group
- ✓ X_1 = Writing to demonstrate knowledge tool
- ✓ X_2 = Performance- based tool
- ✓ X_3 = Reciprocity with peer method
- ✓ X_4 = Project- based method

B. Area of the Study

The area of the study was the South-South Geopolitical Zone of Nigeria which comprises six states, Akwa Ibom, Bayelsa, Cross River, Delta, Edo, and Rivers states. It is strategically located at the point where the Y tail of the river Niger joins the Atlantic Ocean through the Gulf of Guinea. Though a relatively small land, the South-South is the economic mainstay of the country's economy; oil. In addition to oil and gas, the region equally contributes other key resources, with potential huge investment opportunities in tourism and agriculture. There are one hundred and twenty three (123) recognized local government areas and a population of about 23,347,200 people in the area as found in 2016 National Population Census taking approximately 4% of the national annual growth rate into consideration. There are a total of thirteen Colleges of Education in the six states, one in Akwa Ibom, two in Cross River, one in Rivers, one in Bayelsa, four in Delta and four in Edo States.

The South-South Zone is the third largest wetland in the world, after the Mississippi and Pantanal (NDDC, 2013). The Zone covers about 70,000 square kilometers and is noted for its peculiar and difficult terrain. The whole area is traversed and criss-crossed by a large number of rivulets, streams, canals and creeks. The coastal line is buffed through out the year by the tides of the Atlantic Ocean, while the mainland is subjected by regimes of flood by the various rivers, particularly River Niger. There are about thirty-five (35) different ethnic groups speaking about 250 dialects across 4,500 communities, (NDDC, 2013).

The South-South states are the major oil producing states in the country. The people are mainly agrarian and are famous in the cultivation of agricultural crops including yam, rice, palm, cassava, rubber, cocoa, coconuts, and palm oil production, a diversity of aquatic resources and fertile land which supports all year round agriculture. They also venture into art and craft fishing/ farming ceramics/carving and raffia making. The states are also blessed with abundant natural resource including salt deposit, precious stones, crude oil, vast land and forest resources. The region which forms the greater part of the Niger Delta accounts for more than

90% of earnings from oil and gas. It also account for oil resources of about 30 billion barrel and gas reserves of about 160 trillion cubic feet; (NDDC, 2013).

The area is bounded in the East by Imo, Ebonyi and Abia state in the South by Republic of Cameroon, in the West by Ondo State, and in the North by Kogi and Benue States. The climate situation is distinctly marked and influenced by two major seasons: rainy and dry seasons. The vegetation of the area falls within the mangrove forest enclave in the Atlantic Ocean. The relief features of the area is fairly even, with few knolls and valleys that makes it undulating in between. There exist a similar cultural affiliation in terms of dance and traditional festivals among the people of Akwa Ibom and Cross River, while the people of Rivers, Delta, Edo and Bayelsa share some common cultural features. The religion is predominantly Christianity, though there are some practice of African traditional worships in some part of the areas and also some pockets of Moslems among the Hausa settlers. The major occupation of the people of the South- South is farming and fishing.

C. Population of the Study

The population of this study consist of (10,648) year two students in the thirteen Colleges of Education, in the South-South states in the 2018/2019 academic session. The focus of the study was on three thousand three hundred and six (3,306) year two students in Akwa Ibom State College of Education . The choice of year 2 students was based on the fact that they have taken a course on Basic General Mathematics II in their year one and stand a better chance of knowing their capabilities and can know if the intervention is of benefit or not. (See Appendix A pg 229).

D. Sample and Sampling Technique

The sample consisted of (386) students, such that 198 were females and 188 males, 189 were for experimental group while 197 were for control group. The sample size was determined in two stages, using simple random sampling and purposive sampling technique. Purposive sampling technique was used in drawing the one out of thirteen Colleges of Education used for the study (Akwa Ibom State College of Education). Then Purposive sampling technique was use in selecting School of Art and Simple random sampling was used in drawing 386 year two students in the school of Art that offered Basic General Mathematics IV as a general course by balloting method. The choice of this group of students is based on the fact that they have difficulty in learning mathematics, therefore one can really know if the intervention is effective or not. The sample was drawn from two intact classes in the school of art and were assigned to experimental and control groups respectively. The distribution of sample size is shown in Appendix A_{ii} pg 229.

E. Instruments for Data Collection

The study utilized both qualitative and quantitative data collection methods. Creswell and Plano-Clark (2007), states that, “by mixing the datasets the researcher provides a better understanding of the problem than if either dataset had been used alone”. As such, multiple forms of evidence are necessary for educational stake holders to document and inform research problems. The instruments that were used for data collection were Mathematics Achievement Test (MAT), writing prompts (journals, essay, Mathography), Mathematics interest and Self-concept inventory, Project and Alternative-Solutions Worksheets (ASWs). Students were asked to complete an autobiographical essay, self-concept / interest inventory adapted from mathematics self-efficacy and anxiety scales by May (2009), The Interest-A-Lyzer, by Renzulli (1997) at the start of the semester and a reflective essay at the end of the semester. The writing to learn Mathematics based assessment items were developed by the researcher in line with the course outline for year two basic general Mathematics IV.

The experimental group were required to carry out open –ended approach to problems solving, writing that summarizes a unit or chapter, writing that deals with a particular concept in Mathematics, expressive writings and reflections. They also carried out a project in which participants were ask to form their own problem from a given problem relating it to real life problem situation and solved it. Write on the concept of quadratic equation, problem situation that can be solved using the concept that is application of the quadratic equation concept and example of objects that have parabolic shape. The Alternative-Solutions Worksheets (ASWs), performance-based assessment were developed to “encourage students to use “flash back” strategies while finding alternative solutions to Mathematical problems and do a “Think Aloud” to justify the solution. The ASWs have two parts, initial solution part that has the conventional method of solving the problem and alternative solutions part that has the alternative method of solving the problem (see Apendix K, pg 319). These sections were graded equally on a 40 point, 10 points per section, for which an analytic rubric was developed as a guide for evaluation (see Apendix J, pg 318). The control group classes were taught using traditional instruction lecturer method, and assessment grades were dependent upon tests, take home assignments, and class-work. Quantitative data collection instruments that were used for this study included the researcher developed Mathematics Achievement Test (MAT) including: test one, mid-semester test, test three, and the final examination. Test one and three, the mid-semester test, and final examination were trial tested prior to the administration. Multiple revisions were made to all tests based on time allowed, difficulty and the use of a course portfolio. The instructional and assessment instruments for both the experimental and control groups were constructed to reveal the strength and weaknesses in students Mathematical abilities.

F. Validation of Instrument

The writing to learn Mathematics assessment items, the traditional close-ended test items, and the lesson plans were face and content validated by three research experts, one from the Mathematics Department, University of Uyo and two from Measurement

and Evaluation Unit of Michael Okpara University of Agriculture Umudike. In addition the items in the Mathematics Achievement Test were subjected to content validation. The content validation of MAT was done with strict adherence to the test blue print so as to ensure that the test items reflected all details on the test blue print. They were requested to assess the content coverage; the suitability of the items, language used, and item arrangement in logical sequence. The items were scrutinized and those suspected not to be measuring what they were designed to measure were discarded or revised, while others were included. The researcher effected all the corrections given. This procedure guaranteed the validity of the instrument. See Appendix L pg 320.

G. Reliability of the Instrument

Prior to the commencement of the main research intervention, procedures were conducted to establish the inter-rater reliability of the assessment instrument. A pilot study was carried out using writing prompts administered to a sample of forty (40) second-year students from the Department of Social Studies in one of the schools within the study area that was not included in the main research. The rationale for selecting this group was to ensure that the pilot did not interfere with or influence the actual study sample.

For the purpose of scoring the written responses, the researcher adopted a scoring rubric originally developed by Pugalee (2005), which was slightly modified to suit the specific objectives of this study. This rubric was then provided to a panel of trained scorers to ensure uniformity and objectivity in scoring the student responses. Before the scoring exercise, the researcher conducted detailed orientation sessions with each member of the scoring panel on a one-on-one basis. During these sessions, the researcher thoroughly explained the specific elements and indicators to be observed in each writing sample, along with the criteria outlined in the rubric. To ensure clarity and consistency, the researcher also modeled how to score a sample response, explaining each scoring decision according to the rubric's categories.

Following this orientation, each panelist was provided with a student-like sample response and was required to score it independently. This served as a calibration activity to determine whether the panelists could apply the rubric reliably and consistently. This process not only reinforced the panelists' understanding of the rubric but also enabled the researcher to identify any potential inconsistencies or misinterpretations before the main scoring began.

After the main student writing samples were collected during the pilot phase, the researcher took responsibility for scoring all the student responses. In order to assess the inter-rater reliability of the scoring process, two members of the panel were randomly selected and provided with a sample of twenty student responses each. These samples represented approximately 50% of the total responses collected. The panelists independently scored the responses using the same rubric, and their scores were then compared with those of the researcher to evaluate the level of agreement.

The assessment of inter-rater reliability was based on both exact agreement and adjacent agreement. Exact agreement refers to instances where the scores given by the panelists and the researcher matched precisely, while adjacent agreement accounts for slight variations in scoring, such as a one-point difference, which is still acceptable in qualitative scoring. The goal was to achieve at least 85% exact agreement and 100% adjacent agreement, which are considered acceptable thresholds in educational assessment.

To quantify the degree of agreement, the Agreement Rate (AR)—also known as the percentage agreement index—was employed, as recommended by Orwin (1994). The AR is calculated by dividing the number of items on which both scorers agree by the total number of items scored. The first panelist achieved an exact agreement rate of 91% and an adjacent agreement rate of 100%, indicating a high level of scoring consistency. The second panelist recorded an exact agreement rate of 85% and an adjacent agreement rate of 99%. The overall agreement between the researcher and the two panelists yielded an average exact agreement of 88% and 100% adjacent agreement, which confirmed the high reliability of the scoring process for the writing tasks.

In addition to inter-rater reliability, the internal consistency of the Mathematics Achievement Test (MAT) used in the study was also evaluated. To determine the reliability of the instrument, the Cronbach's Alpha (α) coefficient was calculated. According to Ezech (2005), the Cronbach Alpha method is the most appropriate statistical technique for assessing the reliability of instruments that contain polychotomous items, as it measures the degree to which items within a test are consistent with one another. This method also provides estimates for the internal consistency of different subsections of the test as well as the overall reliability of the entire instrument.

By calculating Cronbach's Alpha, the researcher was able to determine the extent to which the various sections of the MAT were measuring the same underlying construct. The resulting values from this analysis supported the internal consistency of the MAT, validating it as a reliable instrument for assessing students' Mathematical achievement in the study. MAT had a reliability coefficient of 0.83. The instruments self-concept / interest inventory was adapted from mathematics self-efficacy and anxiety scales by May (2009), The Interest-A-Lyzer, by Renzulli (1997) and the reliability was re-established using Cronbach Alpha and had a reliability of 0.83 and 0.85 respectively.

H. Method of Data Collection

The data collected for this study included student's journal entries, pre and post mathematics Self-concept and interest inventories and Mathography (Appendix B page 230- 233 C, page 235 and D, page 226). The rationale for using these qualitative

data sources is that they are likely to yield evidence of student mathematical thinking and affective outcome (Merriam 2009; Mertler, 2014). Other data consisted of Mathematics Achievement Test, Alternative-Solutions Worksheets (ASWs) and a performance rubric (Appendix I, page 316) used to assess student's academic achievement and affective outcomes. The instruments used to investigate student's affective outcomes and academic achievement in problem solving are discussed in the following sections.

➤ *Qualitative Data Collection*

Pre and Post mathematics Self-concept/ interest inventories/Mathography and Writing To Demonstrate Knowledge: Students completed a mathematics Self-concept/ interest inventory, composed a Mathematical autobiography called Mathography at the beginning of the research period (See Appendix B pages 230- 233 and C, page 235). The students were asked to write about Mathematics in various ways. These documents were analyzed using Thematic Analysis. Thematic Analysis requires the researcher to focus on identifiable themes and patterns of experiences. Specifically, Template Analysis was used to analyze the text-based data acquired during this study. Template Analysis is the development of a coding "template" which condenses themes determined as significant within the data set by the researcher into meaningful and useful information. In other words, it allows the researcher to make sense of the data. The five specific categories included in the Template Analysis Theme were; Developing problem-solving skills, increasing conceptual understanding, demonstrating procedural application, demonstrating reasoning and developing content connections.

For both the pre and post Self-concept/ interest inventories, students were given a copy and encouraged to respond honestly and without hesitation to the 40-item inventory. These data were used to further investigate student's affective behaviour in the area of Self-concept, attitudinal change, interest, motivation and value in Mathematics. After the pre Self-concept/ interest inventory were collected and scored, the writing based assessment protocol began in the WTLM classes. The performance rubric is broken down into the following problem-solving characteristics mentioned above that is, understands the problem, selects a plan, carry out the plan, communicates and connect the solution to real life situation. Students received scores from 0-4 in each of these five categories. A copy of the performance rubric (Appendix I, page 316 and J, page 318) were given to the students in the experimental group for reference during work time. Mathematics journals were collected at the end of each problem-solving work period, and the problem-solving responses was scored using the performance rubric. Mathematics Journals were returned to the students at the beginning of each problem-solving session with the completed assessment information and lecturers feedbacks. This method of data collection was used to measure student's affects and achievement on each of the problem-solving prompts (see Appendix G and H, pg 312).

➤ *Performance-Based Writing*

An Alternative-Solutions Worksheet (ASW) is a tool that encourages students to come up with alternate methods for solving a problem that borrows from George Polya's four-phase problem solving model which requires students to understand the problem, devise a plan, carry out the plan, and look back at the completed solution, by reconsidering and reexamining the result and the path that led to it. Over a period of twelve weeks, the students in both groups were given two questions to solve at the end of every topic. Each student filled out Open-Ended questions in the ASW and were scored. Students in the experimental group were given guidance to effectively "flash back" than just having the correct alternative solutions, however they were taught to problem solve reflectively, to share their thinking and to listen to the thinking of others in their group. The control group answered the questions as the normal traditional take home assignment method.

Two students were selected every week to be recorded doing a "think aloud" for the group work in the experimental group. Each week the researcher and the raters took field notes while the students are discussing their solutions. Lastly, two focus groups were conducted during the intervention. One took place at the end of the first week of the intervention, and the other took place after the posttest. Field notes, transcriptions of the "think aloud" process and the focus groups, as well as student work were coded using open coding, then analytical coding. Open coding techniques was used to generate descriptive codes in which the researcher categorized the field notes, think-aloud transcripts, and documents into broad topic areas. For example, the researcher identified recurring patterns in the student's work like switching between adding, subtracting, multiplying, and dividing to generate more solutions. After identifying common practices, the researcher looked more specifically at what those practices were and analyzed them further. While analyzing the data, connections to Bloom's Revised Taxonomy (2001) were made. The researcher being a college Mathematics teacher, provided the lens that filtered the way the researcher looked at the student's work, and attempted to figure out their struggling to solve the problems. Bloom's Revised Taxonomy helped in understanding how they used "flash back" techniques. These were scored using a scoring rubrics (see Appendix J, pg 318). The scores for the take home assignment were recorded for the control group students.

➤ *Quantitative Data Collection*

Quantitative data collection was the researcher developed test including: test one, mid-semester test, test three, and the final examination (see Appendix H, pg 312). The participants were first given the test one items as take home assignment (project type or open time method of evaluation) six weeks into the treatment. The responses were collected at the end of 48 hours for scoring. Eight weeks after, all the participants took mid-semester test (reciprocity with peer method of evaluation). During this section, the participants were prevented from any form of consultation of text material, but were allowed to discuss and interact with friends. Ten weeks later, test three, (open-ended test method of evaluation), which they were permitted to bring out their relevant materials

which could be of help in answering the questions, were administered. Again the participants were also prevented from any form of discussion or help from colleagues but were encouraged to consult their textbooks, journals, portfolio and ASWs. At the end of the thirty five minutes, the responses of the participants were collected. Twelve weeks into the semester, examination was administered. The three sets of script were scored alongside scores from writing prompts, and ASW scored using a scoring rubrics (see Appendix I, page 316) were then used for continuous assessment CAS. The CAS and the actual semester examination scores were used for the data analysis. It is believed that utilizing multiple data sources can minimize threats to construct validity. With this in mind, the researcher analyzed student WTLM, conducted focus groups, and have student “think aloud”. These multiple data sources have allowed the researcher to triangulate the data.

➤ *Rubric for Assessing Problem-Solving Processes*

The researcher created and implemented an assessment rubric designed to evaluate five distinct problem-solving dimensions: the development of problem-solving abilities, the enhancement of conceptual understanding, the application of procedures, the demonstration of reasoning, and the establishment of content-related connections. Refer to Appendices I, page 316, for the complete coding guide. To assess a student’s growth in problem-solving abilities while working through a task, the researcher alongside a panel of experts looked for indicators such as whether the participant identified the task’s objective, formulated a comprehensive strategy that reflected full comprehension of the task elements, and executed the plan without mistakes. To assess a student’s progress in conceptual understanding while solving the task, the expert panel examined whether the student recognized and explained key ideas and provided relevant examples or illustrations with explanations where applicable. To measure the student’s procedural application, the panel examined whether the student selected fitting strategies and properly carried them out, and evaluated the appropriateness of both representations and algorithms used.

To evaluate a student’s ability to make Mathematical content connections while solving problems, the researcher and expert team looked for evidence that Mathematical facts were correct, that all relevant concepts and ideas were appropriately acknowledged, and that Mathematical language was correctly employed along with meaningful links to real-life situations. When analyzing Mathematical reasoning, they reviewed whether the student clearly and correctly justified significant steps or processes and supported the validity of the answer with logical explanations. Student scores were calculated by summing the individual scores from all five categories and then dividing the total by 20. For example, if a student received the following ratings: 4 for problem-solving development, 3 for conceptual understanding, 4 for procedural application, 2 for Mathematical content connections, and 3 for Mathematical reasoning, the cumulative score would be 16. Dividing this total (16) by 20 yields 0.8, which, when multiplied by 100, results in a score of 80 percent for that assessment. Full scoring examples and rubric details can be found in Appendices I, J, and K, pages 316 to 319.

➤ *Experimental Procedure*

In the selected Colleges of Education in Akwa Ibom State (AKS), two intact classes each were designated as the experimental and control groups, respectively. The intervention spanned a twelve-week period, equivalent to twelve class sessions for each group, and was conducted during the first semester of the 2018–2019 academic session. The intervention for the experimental group consisted of instruction and assessment through the Writing to Learn Mathematics strategy combined with the Triangulation Method of Assessment, while the control group received conventional instruction and was evaluated using traditional closed-ended test methods.

Both groups underwent the intervention during their regularly scheduled Mathematics periods. At the commencement of the study, a Self-Concept and Interest Inventory alongside a Mathography exercise was administered to both groups as a pre-test. This initial administration took place concurrently in their respective classrooms. One day after the completion of the twelve-week treatment period, the Mathography and survey were re-administered as post-tests. The Mathography required each student to write a short reflective essay narrating their identity as a Mathematics learner. These narratives also explored students' cognitive approaches to mathematical thinking and detailed their previous experiences in Mathematics learning.

Thematic analysis of these pre- and post-treatment essays was used to assess prior experience with writing-to-learn strategies and to establish both the equivalence of the groups at baseline and any emergent differences by the end of the intervention. Students in the experimental group continued to complete multiple writing assignments mid-semester. These included threaded discussions, journal entries, and performance-based assessments which were incorporated into a course portfolio. The “Think Aloud” strategy and alternate-solutions worksheet (a reflective writing tool focused on multiple solution pathways) formed a core part of their engagement. The “Think Aloud” assignments, administered biweekly, aimed to enhance metacognitive awareness by prompting students to articulate their reasoning processes, make connections between topics, identify real-world applications, and develop effective learning strategies.

Approximately six weeks into the semester, both the experimental and control groups took their first test. This was followed by a formal mid-semester test. While both groups studied the same content, the control group did not engage in any reflective or writing-based activities during this period. At the end of the semester, both groups completed a final reflective essay assessing their learning trajectory and self-evaluating their achievement in Mathematics.

Students in the experimental group were specifically asked to reflect on their experience with the Writing to Learn Mathematics strategy and the triangulated assessment method. Several students offered reflections such as “*Writing about how I solved the problems helped me understand better, especially when I had to explain it in my own words*”, and “*At first, it felt strange to write in a math class, but I started seeing how it helped me see my mistakes and fix them*”. These qualitative reflections were instrumental in understanding how writing served as a tool for deeper conceptual engagement and personal growth in Mathematics.

A third test was conducted between the mid-semester and final examinations for both groups. The experimental group also took a cumulative content test, during which they were allowed to refer to their course portfolios due to the integrative nature of the test. However, use of portfolios was not permitted for the first and third tests. The performance of both groups across all three tests was analyzed using inferential statistics to evaluate the impact of the intervention.

To ensure academic equivalence at the beginning of the study, pre-existing differences in mathematical ability were measured using the pre-Mathography and pre-survey scores as covariates. Assessment items used in both pre-tests and post-tests were drawn from selected topics in the Year II Basic General Mathematics IV course outline. Experienced course lecturers from the participating institutions were recruited and trained as research assistants to assist in the delivery of instruction and administration of the intervention.

➤ *Control of Extraneous Variables*

To ensure the reliability and validity of the research findings, several measures were implemented to control for potential extraneous variables:

- *Experimental Bias:*

The researcher took an active role in the implementation of the intervention to eliminate inconsistencies. Specifically, the researcher directly administered the treatment for the experimental group and also taught the control group as a regular course lecturer within the institution. Scoring of all assessment instruments was conducted by the researcher in collaboration with two Mathematics lecturers who were not involved in the intervention delivery. This was done to ensure objectivity and avoid any form of experimental contamination or bias.

- *Lecturer Variability:*

To avoid variability in teaching quality and instructional delivery, the two research assistants selected to assist in the study were Mathematics educators with equivalent academic qualifications and comparable teaching experience. The researcher conducted standardized training sessions for both lecturers, ensuring uniform understanding and consistent implementation of the instructional model for both experimental and control groups. All teaching instruments including the Mathematics learning inventory, assessment items, Mathematics test questions, and lesson plans were prepared by the researcher to ensure consistency in instructional materials.

- *Hawthorne Effect:*

To minimize the Hawthorne effect where students alter their behavior simply because they are aware they are part of a study students were not informed about their participation in an experiment. Both the experimental and control group were taught during normal lecture periods within the existing academic timetable. The researcher, acting as the regular course lecturer, maintained a conventional classroom environment. Lesson content was standardized across both groups to ensure that students' behavior and responses remained natural and unaffected by awareness of the study. This helped maintain the ecological validity of the findings.

In summary, the implementation of the intervention was methodically structured to uphold rigorous research standards while minimizing potential threats to internal validity. Through standardized instruction, blinded scoring, and consistent teaching environments, the researcher ensured that any observed differences in outcomes could be attributed to the instructional method and assessment strategy, rather than extraneous influences.

➤ *Instructional Package for Mathematics*

The instructional packages designed for the Mathematics lessons consisted of structured lesson plans covering selected topics in the Basic General Mathematics IV syllabus. These topics included variation, linear equations, simultaneous equations, quadratic equations, and statistics. For the experimental group, the instruction was delivered using writing-to-learn assessment tools and triangulation assessment strategies, which served both instructional and evaluative purposes. Conversely, the control group was taught using the conventional lecture method, and their assessments were conducted using closed-ended tests.

Reciprocal Teaching, a strategic approach employed in the experimental instruction, has been recognized for its structured design that enhances learners' comprehension, fosters an understanding of complex mathematical concepts, and builds students' confidence and motivation to engage with content. This method comprises five key phases: metacognitive questioning, visualizing, solving, reflecting, and connecting.

At the metacognitive questioning phase, students were instructed to anticipate the nature of the mathematical problems presented to them. They were encouraged to consider which mathematical operations they might need to apply and to predict possible forms of the answer, all based on their prior knowledge. To aid this, students were also directed to use text features such as headings, diagrams, and other contextual cues to guide their thinking.

In the visualization stage, students were guided to classify information into three categories: unfamiliar terms, known facts derived from the problem, and information that still needed to be determined for a solution. This phase required higher-order thinking and often involved collaborative group work to allow learners to exchange insights. After identifying knowledge gaps, students were prompted to revisit the text or problem to re-establish clarity and meaning.

The solving stage marked the actual problem-solving activity. At this point, learners were given access to multiple strategies but were not restricted to a particular method. This autonomy allowed them to choose an approach that best suited their understanding and problem-solving style, thereby promoting learner-centered engagement.

During the reflecting stage, students engaged in personal introspection. They evaluated their own contributions to group discussions, assessed the appropriateness and effectiveness of the strategies they used, and considered how they might improve their approach in future scenarios. Additionally, students were required to justify their answers, encouraging a deeper understanding of the reasoning behind their solutions.

The final stage, connecting, required students to keep written records of their activities and learning outcomes across the previous four stages. This integration of reading and writing supported the reinforcement of key concepts and facilitated the application of these concepts to real-world contexts. The connecting phase is especially crucial as it enhances comprehension, retention, and the transfer of mathematical understanding to practical situations. Furthermore, lecturers provided timely and constructive feedback throughout this phase, which was essential in guiding students' development and refining their skills.

Student reflected on reciprocity with peers in three major ways: 'looking at work,' 'talking,' and 'listening.' Each alternative assessment tools and methods projected, with the lesson plan, incorporated the following strategies:

- Providing conceptual, procedural understanding and affective/reflective outcomes of Mathematical concepts through clear direction to reduce student's confusion.
- Clarifying and simplifying Mathematical concepts through multiple representations for the student understanding.
- Motivating the students interest related to task through visualization of Mathematics concepts.
- Demonstrated guided – discovery learning and experimentation.
- Encouraged students project in Mathematics like manipulation of variables through alternative solution worksheet by using "flash back".
- Students have opportunity to solve problems by investigating Mathematical relations dynamically.

Each of the Alternative and Traditional lesson plans indicated among others, lesson topic, instructional materials and instructional procedures. The instructional procedure showed details of the steps, content development, lecturer's activities, student's activities writing to learn, triangulation and Traditional assessment methods strategies and skills acquired. (Appendix E page 237- 262 and F, page 311).

I. Method of Data Analyses

Data generated were analyzed using mean and standard deviation to answer the research questions. Multivariate Analysis of Covariance (MANCOVA), was used to test hypotheses 1, 2, 3, 6, 7 and 9. Multivariate Analysis of Variance (MANOVA), was used to test hypotheses 8 and Multiple Regression Analysis was used for hypotheses 4 and 5 to determine the relationships between the alternative assessment methods and the overall academic achievement and affective outcomes of the students in mathematics, at 0.05 levels of significance. The Multiple Regression Analysis (M.R.A.) was carried out in order to identify the relative effectiveness of the two tools and techniques of evaluating student's achievement and affective outcomes in mathematics. MANCOVA is a method of data analysis that ensures comparability and equality of groups before treatment. The groups involved will be statistically equated on the basis of critical variable known as covariates (Udo, 2003). In this study, the groups were equated on the basis of pre-inventory and Pre- Mathography scores.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

The purpose of this study was to provide empirical evidence of student's overall academic achievement as well as their mathematics conceptual growth in addition to their meta-cognitive growth by using Writing to Learn Mathematics and triangulation assessment method. This chapter presents the results, interpretation and analysis of data based on the nine research questions and null hypotheses that guided the study. The findings for the research questions are both quantitative and qualitative in nature. The results to these questions are presented in a sequential manner, quantitative then qualitative, although the data were actually collected simultaneously.

➤ Research Question 1:

What is the mean difference between writing to learn Mathematics as an assessment tool and traditional method test in overall academic achievement and affective outcome of students with respect to learning Mathematics?

Table 1 Pretest–Posttest Achievement and Affective Outcomes of Writing-to-Learn and Traditional Methods.

Variables	N	Group	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Achievement	189	Writing to Learn	50.84	78.76	27.92	9.97	9.71
	197	Traditional Method	38.77	43.28	4.51	15.74	12.54
Affective Outcome	189	Writing to Learn	47.12	71.52	24.40	4.41	3.02
	197	Traditional Method	49.60	51.74	2.14	10.04	13.13

Mean and standard deviation was used in answering the question as presented in table one. From table 1, writing to learn assessment method has a mean gain score of 27.92 in academic achievement while traditional method has a mean gain score of 4.51. The mean gain score of writing to learn method for affective outcome was 24.40 while that of the traditional method was 2.14. This result shows that the writing to learn assessment method increased both academic achievement and affective outcome of students in learning of Mathematics. The mean of Pre- Achievement Test was (\bar{X} =50.84 SD= 9.97) and Pre-Affective outcome was (\bar{X} = 47.12, SD =4.41) for Writing to learn method while the mean of Pre- Achievement Test was (\bar{X} =38.77, SD=12.54) and Pre-Affective outcome (\bar{X} =49.60, SD= 13.13) for Traditional method. The means of Writing to learn method group in Pre-Test, were almost the same with that of the Traditional method group, and the high standard deviation in both group shows that the groups were the same before treatment.

➤ Hypothesis 1:

There is no significant mean difference between writing to learn Mathematics as an assessment tool and overall academic achievement and affective outcome of students with respect to learning Mathematics.

Table 2 Result of MANCOVA – Tests of Between Writing-to-Learn Assessment Tool and Traditional Assessment Method

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	ACADA_POSTTEST	95,025.242	2	31,675.08	243.21	.00
	AFFECT_POSTTEST	46,709.349	2	15,569.78	1,146.31	.00
Intercept	ACADA_POSTTEST	2,177.585	1	2,177.59	16.72	.00
	AFFECT_POSTTEST	3,622.869	1	3,622.87	266.73	.00
ACADA_PRETEST	ACADA_POSTTEST	5,383.426	1	5,383.43	41.34	.00
	AFFECT_POSTTEST	2.183	1	2.18	0.16	.69
AFFECT_PRETEST	ACADA_POSTTEST	130.340	1	130.34	1.01	.32
	AFFECT_POSTTEST	385.530	1	385.53	28.38	.00
ASSESSMENT METHOD	ACADA_POSTTEST	24,301.373	1	24,301.37	186.60	.00
	AFFECT_POSTTEST	22,608.333	1	22,608.33	1,664.51	.00
Error	ACADA_POSTTEST	49,750.143	383	130.24		
	AFFECT_POSTTEST	5,188.540	383	13.58		
Total	ACADA_POSTTEST	1,446,181.000	386			
	AFFECT_POSTTEST	1,457,021.000	386			
Corrected Total	ACADA_POSTTEST	144,775.381	385			
	AFFECT_POSTTEST	51,897.889	385			

a. R Squared = .656 (Adjusted R Squared = .654) b. R Squared = .900 (Adjusted R Squared = .899)

The result in table 2 revealed a significant main effect was observed for assessment tools $F(1, 382) = 186.60$ and 1664.51 , $p < 0.00$ for achievement and affective outcome respectively. To specify the direction of the effect a post hoc multiple comparison of post-test mean achievement and affective outcome was carried out. This is presented in table 3.

Table 3 Pairwise Comparison of Writing-to-Learn and Traditional Methods

Dependent Variable	(I) Assessment Method	(J) Assessment Method	Mean Difference (I-J)	Sig.
Achievement	Writing to Learn	Traditional Method	22.70*	.00
Achievement	Traditional Method	Writing to Learn	-22.70*	.00
Affective Outcome	Writing to Learn	Traditional Method	21.90*	.00
Affective Outcome	Traditional Method	Writing to Learn	-21.90*	.00

Note. $p < .05$.

Table four point zero three reveals that significant difference exist between the achievement scores of students assessed with writing to learn method and those assessed with the traditional method in favour of the writing to learn method. A significant difference was also observed for the affective outcome scores between the writing to learn method and traditional method in favour of writing to learn method. This shows that the implementation of writing to learn assessment tools increased students academic achievement and affective outcome more than the traditional method of assessment.

➤ *Research Question 2:*

What is the mean difference between triangulation approach as assessment method and traditional test method in overall academic achievement and affective outcome of students with respect to learning Mathematics?

Table 4 Pretest–Posttest Achievement and Affective Outcomes of Triangulation and Traditional Methods

Variables	N	Group	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Achievement	189	Triangulation	43.10	73.60	30.50	11.54	8.78
	197	Traditional Method	38.77	43.28	4.51	15.74	12.54
Affective	189	Triangulation	47.12	71.52	24.40	4.41	3.02
Outcome	197	Traditional Method	49.60	51.74	2.14	10.04	13.13

Mean and standard deviation was used in answering this question as presented in table 4. From table 4, Triangulation of assessment method has a mean gain score of 30.50 in academic achievement while traditional method has a mean gain score of 4.51. The mean gain score of triangulation method for affective outcome was 24.40 while that of the traditional method was 2.14. This result shows that the Triangulation of assessment method increased both academic achievement and affective outcome of students in learning of Mathematics. The mean of Pre- Achievement Test was (\bar{X} =43.10SD= 8.78) and Pre-Affective outcome was (\bar{X} = 47.12, SD =4.41) for Triangulation method while the mean of Pre- Achievement Test was (\bar{X} =38.77, SD=12.54) and Pre- Affective outcome (\bar{X} =49.60, SD= 13.13) for Traditional method. The means of Triangulation method group in Pre-Test, were almost the same with that of the Traditional method group, the high standard deviation for both methods of assessments shows that the groups were the same before treatment.

➤ *Hypothesis 2:*

There is no significant mean difference between triangulation approach as assessment method and traditional test in overall academic achievement and affective outcome of students with respect to learning Mathematics.

Table 5 Result of MANCOVA – Tests of Between Triangulation Assessment and Traditional Assessment Methods.

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	ACADA_POSTTEST	92,663.638	2	30,887.88	226.42	.00
	AFFECT_POSTTEST	46,790.443	2	15,596.81	1,166.53	.00
Intercept	ACADA_POSTTEST	2,509.287	1	2,509.29	18.39	.00
	AFFECT_POSTTEST	3,808.134	1	3,808.13	284.82	.00
ACADA_PRETEST	ACADA_POSTTEST	3,021.821	1	3,021.82	22.15	.00
	AFFECT_POSTTEST	83.277	1	83.28	6.23	.01
AFFECT_PRETEST	ACADA_POSTTEST	216.625	1	216.63	1.59	.21
	AFFECT_POSTTEST	386.423	1	386.42	28.38	.00
ASSESSMENT METHOD	ACADA_POSTTEST	80,341.297	1	80,341.30	588.93	.00
	AFFECT_POSTTEST	44,663.903	1	44,663.90	3,340.54	.00
Error	ACADA_POSTTEST	52,111.743	382	136.42		
	AFFECT_POSTTEST	5,107.446	382	13.37		
Total	ACADA_POSTTEST	1,446,181.000	386			
	AFFECT_POSTTEST	1,457,021.000	386			
Corrected Total	ACADA_POSTTEST	144,775.381	385			
	AFFECT_POSTTEST	51,897.889	385			

a. R Squared = .656 (Adjusted R Squared = .654)b. R Squared = .900 (Adjusted R Squared = .899)

The result in table 5 revealed a significant main effect was observed for assessment methods $F(1, 382) = 588.93$ and 3340.54 , $p < 0.00$ for achievement and affective outcome respectively. To specify the direction of the effect a post hoc multiple comparison of post-test mean achievement and affective outcome was carried out. This is presented in table 6.

Table 6 Post Hoc Multiple Comparisons of Posttest Means Achievement and Affective outcome.

Dependent Variable	(I) Assessment Method	(J) Assessment Method	Mean Difference (I-J)	Sig.
Achievement	Writing to Learn	Traditional Method	22.70*	.00
Achievement	Traditional Method	Writing to Learn	-22.70*	.00
Affective Outcome	Writing to Learn	Traditional Method	21.90*	.00
Affective Outcome	Traditional Method	Writing to Learn	-21.90*	.00

Note. $p < .05$. Based on observed means, the error term is Error.

Table 6 reveals that significant difference exist between the achievement scores of students assessed with triangulation method and those assessed with the traditional method in favour of the triangulation method. A significant difference was also observed for the affective outcome scores between the triangulation method and traditional method in favour of triangulation method. This shows that the implementation of triangulation of assessment methods increased students academic achievement and affective outcome more than the traditional method of assessment.

➤ Research Question 3:

What is the mean difference in student's overall academic achievements, Self-concept and interest (affective outcomes) between the alternative assessment group and the traditional assessment group in Mathematics?

Table 7 Pretest–Posttest Achievement and Affective Outcomes of Experimental and Control Groups

Variables	N	Group	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Achievement	189	Experimental	50.85	73.60	22.76	11.54	9.97
	197	Control	38.82	43.27	4.51	15.77	12.57
Affective	189	Experimental	47.12	72.73	25.61	6.47	3.02
Outcome	197	Control	46.56	49.60	3.04	3.13	3.35

Mean and standard deviation was used in answering this question as presented in table 7. From table 7, The mean gain scores in achievement and affective outcome (22.76/25.61) of experimental group is more than academic achievement and affective outcome mean gain scores (4.51/3.04) of the control group. The mean gain score of triangulation method for affective outcome was 24.40 while that of the traditional method was 2.14. This result shows that the usage of Triangulation of assessment method and writing to learn increased both academic achievement and affective outcome of students in the learning of Mathematics. However the experimental group students scores in achievement and affective outcome spread more ($SD = 11.54/9.97, 6.47/3.02$) before treatment and less after treatment while the control group students scores in achievement and affective outcome spread less ($SD = 12.57/15.77, 3.13/3.35$) before treatment and more after treatment.

➤ Hypothesis 3:

There is no significant mean difference in student's overall academic achievements, Self-concept and interest(affective outcomes) between the alternative assessment group and the traditional assessment group in Mathematics.

Table 8 Result of MANCOVA – Tests of Between Experimental and Control Groups

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	ACADA_POSTTEST	93,149.846	2	31,049.95	229.75	.00
	AFFECT_POSTTEST	46,707.585	2	15,569.20	1,145.87	.00
Intercept	ACADA_POSTTEST	2,152.425	1	2,152.43	15.93	.00
	AFFECT_POSTTEST	3,535.926	1	3,535.93	260.24	.00
ACADA_PRETEST	ACADA_POSTTEST	3,508.030	1	3,508.03	25.96	.00
	AFFECT_POSTTEST	0.419	1	0.42	0.03	.86
AFFECT_PRETEST	ACADA_POSTTEST	203.725	1	203.73	1.51	.22
	AFFECT_POSTTEST	383.872	1	383.87	28.25	.00
ASSESSMENT METHOD	ACADA_POSTTEST	25,749.513	1	25,749.51	190.53	.00
	AFFECT_POSTTEST	21,453.952	1	21,453.95	1,578.99	.00
Error	ACADA_POSTTEST	51,625.540	382	135.15		
	AFFECT_POSTTEST	5,190.300	382	13.59		
Total	ACADA_POSTTEST	1,446,181.000	386			
	AFFECT_POSTTEST	1,457,021.000	386			
Corrected Total	ACADA_POSTTEST	144,775.381	385			
	AFFECT_POSTTEST	51,897.889	385			

a. R Squared = .656 (Adjusted R Squared = .654) b. R Squared = .900 (Adjusted R Squared = .899)

The null hypothesis tested was at 0.05 level of significance. The hypothesis was tested to find out whether the difference in mean found in research questions is significant or not. Also the hypothesis three was used to find out the interaction effects of alternative assessment methods and traditional assessment method as measures of increase in academic achievement and affective outcome of students as measured by mean achievement and affective outcome scores in the Post-test. The result of the analysis is presented in Table 8.

As it is seen in Table 8, there was a statistically significant mean difference between experimental and control group $F(1,382) = 19.05$, $P < 0.00$ and $F(1, 382) = 15.79$ $P < 0.00$ in respect to post-test scores of academic achievement and affective outcome. This means that writing to learn mathematics and triangulation of assessment method usage increased student academic achievement and affective outcome in the learning of mathematics positively.

➤ *Research Question 4:*

How does the nature of student's individual meta-cognitive functioning (Self-concept, interest, attitudinal change, motivation and value) increased academic achievement in the learning of Mathematics?

Table 9 Pretest–Posttest Means of Affective Dimensions (Experimental Group)

Variables	N	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Self-concept	189	46.38	70.72	24.34	4.93	3.22
Interest	189	47.25	72.99	25.74	5.31	2.80
Attitudinal Change	189	47.57	72.16	24.59	4.89	2.37
Motivation	189	47.59	72.63	25.04	6.07	2.88
Value	189	46.31	72.22	25.91	5.77	3.33

To answer research question 4, a template was developed from questions asked of students on their first writing excerpt for the course. All students completed a Learning Biography the first week of the course which addressed: who they are as students, feelings they have toward mathematics in general, their strengths and weaknesses as mathematics students, goals for this course, long term educational goals, and why the student felt he/she would be successful in the course, method of assessment used by the lecturers and how it has helped them to learn mathematics which was measured under Self-concept, interest, attitudinal change, motivation and value as sub-headings. Data extraction initially demonstrated a fairly superficial nature of reflection. The majority of the Learning Biographies did not address all of the questions asked, lacked specific detail, and lacked substantive Self-concept. The nature of meta-cognitive functioning was artificial or superficial and low level of meta-cognitive functioning. This is revealed in the mean and standard deviation of the pre-test scores (46.38/47.25/47.57/47.59 and 46.31). The low level nature of meta-cognitive functioning improved after the intervention to high level, substantial and deeper meta-cognitive functioning. This was reflected in the high mean gain and well spread standard deviation of the post-test scores.

➤ *Hypothesis 4:*

There is no significant relationship between the nature of student's individual meta-cognitive functioning (Self-concept, interest, attitudinal change, motivation and value), and increased academic achievement in the learning of Mathematics.

Table 10 Model Summary for Regression Analysis

Model	Sum of Squares	df	Mean Square	F	Sig.	R	R ²	Adjusted R ²	SEM
Regression	1,025,167.18	5	205,033.44	1,588.75	.00	0.99	0.98	0.98	11.36
Residual	23,745.82	184	129.05						
Total	1,048,913.00	189							

A multiple regression was carried out to investigate whether the nature of Self-concept, interest, attitudinal change, motivation and value of mathematics learning could significantly predict participants' overall academic achievement in mathematics. The results of the regression indicated that the model explained 97.7% of the variance and that the model was a significant predictor of academic achievement, $F(5,184) = 1588.75$, $p = .000$. While Self-concept, motivation, value and interest contributed more significantly to the model ($B = .67$, $.66$, $.53$ and $.51$, $p < .05$), attitudinal change contributed less significantly to the model ($B = .42$, $p = .03$). The final predictive model was: Academic achievement = $20.66 + (.67 * \text{Self-reflection}) + (.51 * \text{Interest}) + (.42 * \text{Attitudinal change}) + (.66 * \text{Motivation}) + (.53 * \text{Value})$. For every additional point achieved on Self-concept, Interest, Attitudinal change, Motivation and Value we can interpret that the Academic achievement Score increases by 0.67, 0.51, 0.42, 0.66 and 0.53.

Table 11 Coefficients of Predictors of Academic Achievement (Regression)

Variable	B	SE	Beta	t	Sig.
Self-concept	.67	.28	.41	2.34	.03
Interest	.51	.19	.50	2.77	.01
Attitudinal Change	.42	.19	.41	2.15	.03
Motivation	.66	.15	.32	2.33	.00

Value	.53	.12	.32	4.32	.00
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Since we have multiple independent variables in the analysis the Beta weights compare the relative importance of each independent variable in standardized terms We find that Self-concept has the highest impact followed by motivation, interest and value while attitudinal change had the lowest impact (beta = .67,.66,.53, .51 and beta = .42 the lowest).

➤ *Research Question 5:*

How does meta-cognitive functioning change, (Self-concept, interest, attitudinal change, motivation and value) effect academic achievement of students in the learning of Mathematics?

To demonstrate the nature of student's individual meta-cognitive functioning and the ways in which it may have changed during the course of this study, answering research question five, further examination of student's quotes from the constructs of changes as a learner, Self-concepts, interest and value of writing were necessary to highlight distinguishable differences and insights amongst the participants in the study. These three constructs focused on the nature of student's individual meta-cognitive functioning and allowed the researcher to examine how student's individual Meta-cognitive functioning changed during the course of the study. Within the changes as a learner construct, it was clear when students stated experiences that represented substantive changes or did not seem to experience any changes as a result of the intervention, Writing to Learn Mathematics and triangulation assessment method. The result in table 9 showed a mean gain of 24.34, 25.74, 24.59 and 25.91 for self –concept, interest, attitudinal change, motivation and value in the learning of mathematics. This shows that there was a substantive, high level, deeper nature of meta-cognitive functioning change as students engaged in the intervention.

➤ *Hypothesis 5:*

There is no significant relationship between the meta-cognitive functioning changed (Self-concept, interest, attitudinal change, motivation and value) and academic achievement in the learning of Mathematics.

Multiple regressions was employed to assess how much variance can be explained by the student's individual meta-cognitive functioning change (by the independent variables, Self-concept, interest, attitudinal change, motivation and value of mathematics learning). The result of the multiple regressions or the adjusted R square in Table 9 indicates that 98 percent of the variance in academic achievement is attributed to the variance of the combination of the five independent variables (i.e. , self-concept, interest, attitudinal change, motivation and value of mathematics learning). Moreover, F-test was employed to determine whether the result is statistically significant or not. The result showed that the correlation between the dependent variable (academic achievement) and the combination of the five independent variable (Self-concept, interest, attitudinal change, motivation and value of mathematics learning) was statistically significant at $F(5,184) = 1588.75$, $p = 0.000$. The results of the regression indicated that the model explained 97.7% of the variance and that the model was a significant predictor of academic achievement, $F(5,184) = 1588.75$, $p = .00$. While Self-concept, motivation, value and interest contributed more significantly to the model ($B = .67, .66, .53$ and $.51$, $p < .05$), attitudinal change contributed less significantly to the model ($B = .42$, $p = .03$). The final predictive model was: Academic achievement = $20.66 + (.67 * \text{Self-concept}) + (.51 * \text{Interest}) + (.42 * \text{Attitudinal change}) + (.66 * \text{Motivation}) + (.53 * \text{Value})$. This result shows that the students meta-cognitive functioning changed after the intervention, therefore hypothesis 5 which states that there is no significant relationship between the meta-cognitive functioning changed (Self-concept, interest, attitudinal change, motivation and value) and academic achievement in the learning of Mathematics was not accepted.

➤ *Research Question 6:*

What is the mean difference between the assessment approach reciprocity with peer and project- based methods of evaluating and increase in academic achievement and affective outcome of students in the learning of Mathematics?

Table 12 Pretest–Posttest Achievement and Affective Outcomes for Reciprocity and Project-Based Strategies

Variables	N	Group	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Reciprocity	189	Strategy	40.26	74.42	34.16	12.50	11.77
Project-based	189	Strategy	42.42	74.39	31.97	13.19	7.83
Achievement	189	Combined	42.87	73.60	30.73	9.41	6.07
Affective Outcome	189	Combined	47.12	72.22	25.91	7.34	3.33

As it can be seen from table 12, Reciprocity had the highest mean gain score 34.16 followed by Project-based method 31.97 that had the lowest mean gain score. But a closer look at the mean gain score shows that the difference between the methods is a chance event thus, the two methods achieved equally. From table 12 above also the mean for pretest for reciprocity is 40.26 and project-based is 42.42 with standard deviation of 12.50 and 13.19. Their pre-test mean scores and standard deviation for achievement and affective outcome were (42.87/9.41, 47.12/7.34) respectively. This implies that at the commencement of this study, the subjects were at the same level in the knowledge of and affective behaviour in mathematics. However, in the post-test as seen in table 12 the mean scores increased at almost the same level 74.42 and 74.39. This shows that the use of triangulation of assessment method have effect on student's achievement and affective outcome positively and equally.

➤ *Hypothesis 6:*

There is no significant mean difference between the assessment approach reciprocity with peer and project techniques of evaluation and increase in academic achievement and affective outcome of students in the learning of Mathematics.

Table 13 Result of MANCOVA – Tests of Between Experimental and Control Groups

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	ACADA_POSTTEST	93,149.846	2	31,049.95	229.75	.00
	AFFECT_POSTTEST	46,707.585	2	15,569.20	1,145.87	.00
Intercept	ACADA_POSTTEST	2,152.425	1	2,152.43	15.93	.00
	AFFECT_POSTTEST	3,535.926	1	3,535.93	260.24	.00
ACADA_PRETEST	ACADA_POSTTEST	3,508.030	1	3,508.03	25.96	.00
	AFFECT_POSTTEST	0.419	1	0.42	0.03	.86
AFFECT_PRETEST	ACADA_POSTTEST	203.725	1	203.73	1.51	.22
	AFFECT_POSTTEST	383.872	1	383.87	28.25	.00
ASSESSMENT METHOD	ACADA_POSTTEST	25,749.513	1	25,749.51	190.53	.00
	AFFECT_POSTTEST	21,453.952	1	21,453.95	1,578.99	.00
Error	ACADA_POSTTEST	51,625.540	382	135.15		
	AFFECT_POSTTEST	5,190.300	382	13.59		
Total	ACADA_POSTTEST	1,446,181.000	386			
	AFFECT_POSTTEST	1,457,021.000	386			
Corrected Total	ACADA_POSTTEST	144,775.381	385			
	AFFECT_POSTTEST	51,897.889	385			

a. R Squared = .674 (Adjusted R Squared = .669) b. R Squared = .410 (Adjusted R Squared = .402)

From table 13, a significant main effect was observed for assessment methods with respect to post-achievement and post-affective outcome $F(1,37)=147.15$, $P<0.00$ $F(1,37) = 206.21$, $p<0.00$. The multivariate component of the MANCOVA revealed a significant effect of total triangulation assessment score, Wilks' Lambda $F(1,37) = 26.43$, $p<.00$. That is, controlling for the covariates in the model, the pre-reciprocity, and project-based assessment methods on the triangulation assessment methods remained a significant predictor of scores on the dependent variables for the academic achievement and affective outcome. To specify the direction of the effect a post hoc multiple comparison of post-test mean achievement and affective outcome was carried out. This is presented in table 14.

Table 14 Pairwise Comparison of Reciprocity and Project-Based Assessment Methods

Dependent Variable	(I) Assessment Method	(J) Assessment Method	Mean Difference (I-J)	Sig.
Achievement	Reciprocity	Project-based	12.51*	.00
Achievement	Project-based	Reciprocity	-12.51*	.00
Affective Outcome	Reciprocity	Project-based	0.04	1.00
Affective Outcome	Project-based	Reciprocity	-0.04	1.00

Note. $p < .05$, Based on observed means, the error term is Error. b. Adjustment for multiple comparisons: Bonferroni.

Table 14 reveals that significant difference exist between reciprocity method and project-based method in favour of reciprocity. The post hoc reveal that reciprocity methods contributed more to the increase in academic achievement of the students in mathematics while project-based contributed less. Table 14 also reveals that all two assessment methods are equally effective in increasing affective outcome of students in the learning of mathematics.

➤ *Research Question 7:*

What is the mean difference between the assessment tools, writing to demonstrate knowledge and performance- based and increase in academic achievement and affective outcome of students in the learning of Mathematics?

Table 15 Pretest–Posttest Achievement and Affective Outcomes of Writing to Demonstrate and Performance-Based Assessment

Variables	N	Group	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Achievement	189	Writing to Demonstrate	48.41	78.89	30.48	10.41	9.67
	189	Performance-based	46.68	78.04	31.36	13.83	10.39
Achievement (Comb.)	189	Overall	42.87	73.60	30.73	9.41	6.07
Affective Outcome	189	Combined	47.12	72.22	25.91	7.34	3.33

As it can be seen from table 15, Performance-based tool had the highest mean gain score 31.36 and writing to demonstrate knowledge tool 30.48 had the lowest mean gain score. But a closer look at the mean gain score shows that the difference between the methods is a chance event thus, the two methods achieved equally. From table 15 above also the mean for pretest for writing to demonstrate knowledge is 48.41, and performance-based is 46.68 with standard deviation of 10.41 and 13.83. Their pre-test mean scores and standard deviation for achievement and affective outcome were (42.87/9.41, 47.12/7.34) respectively. This implies that at the commencement of this study, the subjects were at the same level in the knowledge of mathematics. However, in the post-test as seen in table 15 the mean scores increased at almost the same level 78.89 and 78.04. This shows that the use of writing to learn tools to assess, improved on student's achievement and affective outcome positively and equally.

➤ *Hypothesis 7:*

There is no significant mean difference between the assessment tools, writing to demonstrate knowledge and performance-based tool and increase in academic achievement and affective outcome of students in the learning of Mathematics.

Table 16 Result of MANCOVA – Tests of Between Writing-to-Learn Assessment Tools, Academic Achievement, and Affective Outcome in Mathematics

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	ACADA_POSTTEST	5,788.45	3	1,447.11	16.38	.00
	AFFECT_POSTTEST	1,990.17	3	497.54	9.97	.00
Intercept	ACADA_POSTTEST	8,190.26	1	8,190.26	92.69	.00
	AFFECT_POSTTEST	3,898.10	1	3,898.10	78.12	.00
ACADA_PRETEST	ACADA_POSTTEST	1.10	1	1.10	0.01	.00*
	AFFECT_POSTTEST	1,341.50	1	1,341.49	26.88	.01
AFFECT_PRETEST	ACADA_POSTTEST	47.87	1	47.87	0.54	.00*
	AFFECT_POSTTEST	324.64	1	324.64	6.51	.00
WDK_PRETEST	ACADA_POSTTEST	4,932.31	1	4,932.31	55.82	.00
	AFFECT_POSTTEST	8.15	1	8.15	0.16	.68
PERFORMANCE_PRETEST	ACADA_POSTTEST	11.93	1	11.93	0.23	.63
	AFFECT_POSTTEST	0.01	1	0.01	0.00	.98
WRITING_METHOD	ACADA_POSTTEST	1,208.37	1	1,208.37	13.68	.00
	AFFECT_POSTTEST	69.05	1	69.05	1.38	.04
Error	ACADA_POSTTEST	32,958.01	374	88.36		
	AFFECT_POSTTEST	4,305.17	374	49.91		
Total	ACADA_POSTTEST	1,143,099.00	378			
	AFFECT_POSTTEST	1,940,730.00	378			
Corrected Total	ACADA_POSTTEST	59,027.44	377			
	AFFECT_POSTTEST	7,298.37	377			

a. R Squared = .674 (Adjusted R Squared = .669) b. R Squared = .410 (Adjusted R Squared = .402)

From table 16, a significant main effect was observed for assessment methods with respect to post-achievement and post-affective outcome $F(1,37) = 13.68, P < 0.00$ $F(1,37) = 1.38, p < 0.04$. The multivariate component of the MANCOVA revealed a significant effect of total Writing to learn assessment score, Wilks' Lambda $F(1,37) = 8.79, p < .000$. That is, controlling for the covariates in the model, the pre-writing to demonstrate knowledge and performance-based assessment tools on the writing to learn assessment methods remained a significant predictor of scores on the dependent variables for the academic achievement and affective outcome. To specify the direction of the effect a post hoc multiple comparison of post-test mean achievement and affective outcome was carried out. This is presented in table 17.

Table 17 Post Hoc Multiple Comparisons of Posttest Means Achievement and Affective outcome.

Dependent Variable	(I) Assessment Method	(J) Assessment Method	Mean Difference (I-J)	Sig.
Achievement	WDK	Performance-based	3.560*	.00
Achievement	Performance-based	WDK	-3.560*	.00
Affective Outcome	WDK	Performance-based	-1.638	.08
Affective Outcome	Performance-based	WDK	1.638	.08

Note. $p < .05$. Based on observed means, the error term is Error. b. Adjustment for multiple comparisons: Bonferroni.

Table 17 reveals that significant difference exist between writing to demonstrate knowledge and performance-based tools in favour of writing to demonstrate knowledge. However, the post hoc reveal that writing to demonstrate knowledge tools contributed more to the increase in academic achievement of the students in mathematics while project-based contributed less. Table 17 also reveals that all two assessment methods are equally effective in increasing affective outcome of students in the learning of mathematics.

➤ *Research Question 8:*

What is the mean interaction effect of writing to learn as an assessment tool and triangulation assessment approach as method of evaluating academic achievement and affective outcome of students in the learning of Mathematics?

Table 18 Pretest–Posttest Achievement and Affective Outcomes of Writing to Demonstrate and Performance-Based Assessment

Variables	N	Group	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Achievement	189	Writing to Demonstrate	48.41	78.89	30.48	10.41	9.67
	189	Performance-based	46.68	78.04	31.36	13.83	10.39
Achievement (Comb.)	189	Overall	42.87	73.60	30.73	9.41	6.07
Affective Outcome	189	Combined	47.12	72.22	25.91	7.34	3.33

Table 18 illustrates the mean scores, standard deviation, and sample size for each independent variables and dependent variables. The result indicates that both writing to learn tools and triangulation methods showed minor difference in their mean scores. However, the observed mean score of the two groups in the overall academic achievement scores (Writing to learn= 78.47, Triangulation = 74.41) showed perceptible differences but was the same for affective outcome.

➤ *Hypothesis 8:*

There is no significant interaction effect of writing to learn as an assessment tool and the triangulation assessment approach as method of evaluating academic achievement and affective outcome of students in the learning of Mathematics.

Table 19 MANCOVA Results: Tests of Between-Subjects Effects

Source	Dependent Variable	SS (Type III)	df	MS	F	Sig.	Partial Eta ²
Corrected Model	Achievement	42,036.86	3	14,012.29	109.35	.00	.30
	Affective Outcome	23,892.20	3	7,964.07	92.56	.00	.27
Intercept	Achievement	4,106,770.37	1	4,106,770.37	32,047.26	.00	.98
	Affective Outcome	4,314,222.35	1	4,314,222.35	50,140.99	.00	.99
Triangulation Method	Achievement	7,615.75	1	7,615.75	147.13	.00	.60
	Affective Outcome	2,386.42	1	2,386.42	206.21	.00	.04
Writing Method	Achievement	1,208.37	1	1,208.37	13.68	.00	.03
	Affective Outcome	69.05	1	69.05	1.38	.04	.05
Triangulation * Writing Method	Achievement	14,310.62	3	14,012.29	109.35	.53	.62
	Affective Outcome	14,000.42	3	7,964.07	92.56	.61	.61
Error	Achievement	96,366.77	752	128.15			
	Affective Outcome	64,703.45	752	85.04			
Total	Achievement	4,245,174.00	756				
	Affective Outcome	4,402,818.00	756				
Corrected Total	Achievement	138,403.63	755				
	Affective Outcome	88,595.55	755				

a. R Squared = .750 (Adjusted R Squared = .733) b. R Squared = .093 (Adjusted R Squared = .031)

A Two-way MANOVA was conducted to determine if significant interaction effect existed between the independent variables of Writing to learn assessment tools and Triangulation assessment methods and the two dependent variables, academic achievement and affective outcome. There was no interaction effect existing between the independent variables of Writing to learn assessment tools and Triangulation assessment methods and the two dependent variables, academic achievement and affective outcome. An analysis of the F statistic using the reported Wilks's lambda values was conducted to determine the existence of a significant main effect for Writing to learn assessment tools and Triangulation assessment methods (The Wilks's lambda test was selected to evaluate the variance which is not explained by the two independent variables) for this model Significant main effects were found for both Writing to learn assessment tool and Triangulation assessment method, Wilk's lambda = 0.51, $F(3, 752) = 109.35$, $p = 0.00$, Partial eta squared = 0.31 for academic achievement and affective outcome as seen in Table 19.

The significant multivariate effects for Writing to learn assessment tools and Triangulation assessment methods indicates their impact on the two dependent variables. A final analysis was required to analyze each dependent variable for its effect on the respective groups of the two independent variables. The Test of Between-Subjects Effects was used to determine statistical

significance between Writing to learn assessment tools and Triangulation assessment methods and each of the two dependent variables.

Table 20 Wilks' Lambda for the Multivariate Test Comparing Writing to Learn Tools and Triangulation Assessment Methods

Effect	Test Statistic	Value	F	Hypothesis df	Error df	Sig.	Partial Eta ²
Intercept	Wilks' Lambda	.01	36,741.20	2.00	751.00	.00	.99
Assessment	Wilks' Lambda	.51	101.04	6.00	1502.00	.00	.29

a. Design: Intercept + Assessment _Methods

b. Exact statistic.

Table 20 reports the findings utilized for the final analysis. For writing to learn in academic achievement ($p < .00$) for both methods. Students in writing to learn ($M = 78.89$ and 78.04 , $SD = 9.67$, 9.64) reported higher scores on their academic achievement scores than in Triangulation methods ($M = 74.42$, 74.39 , $SD = 11.77$ and 7.83). It follows that writing to learn contributed more to academic achievement, but the two independent variable contribute equally to affective outcome.

➤ *Research Question 9:*

What is the mean difference in male and female student's overall academic achievements and affective outcomes between the experimental and the control groups in Mathematics?

Table 21 Pretest–Posttest Achievement and Affective Outcomes by Gender (Experimental and Control Groups)

Variables	N	Group	Gender	Pre-test Mean	Post-test Mean	Mean Gain	SD (Pre-test)	SD (Post-test)
Achievement	98	Experimental	Female	40.05	73.83	33.78	10.44	8.22
	91	Experimental	Male	40.32	69.00	28.68	9.99	10.42
	100	Control	Female	47.30	53.70	6.40	9.97	14.29
	97	Control	Male	49.34	58.87	9.53	10.69	9.06
Affective	98	Experimental	Female	47.64	72.73	25.09	9.32	7.38
Outcome	91	Experimental	Male	47.72	71.64	23.92	8.02	9.47
	100	Control	Female	47.02	51.78	4.76	13.97	15.28
	97	Control	Male	46.56	56.60	10.0		

As it can be seen in table 21, the mean gain scores (33.78, 25.09) in academic achievement and affective outcome of female in experimental group are higher than that of their male counterpart (28.68, 23.92), while in control group the reverse is the case. The overall academic achievement and affective outcome (73.82, 72.73) of female students in experimental group are higher than that of male (69.00, 71.64), (58.87, 56.60) in both experimental and control group as well as that of the female (53.70, 51.78) in the control group. Also the spread of the academic achievement and affective outcome of female was higher than that of the male before treatment but the reverse was the case after treatment this shows that the implementation of the alternative assessment method was able to reduced the deviation in scores of the female group from 10.44 to 8.22, while the male scores spread better than that of the female 10.69 to 9.06 in the control group.

➤ *Hypothesis 9:*

There is no significant mean difference in male and female student's overall academic achievements and affective outcomes between the experimental and the control groups in Mathematics.

Table 22 Result of MANCOVA – of Male and Female Students' Academic Achievement and Affective Outcome of Experimental and Control Groups in Mathematics

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	ACADA_POSTTEST	82520.95a	5	16504.19	136.04	.00
	AFFECT_POSTTEST	56178.10b	5	11235.62	1142.17	.00
Intercept	ACADA_POSTTEST	1420.53	1	1420.53	11.71	.00
	AFFECT_POSTTEST	2641.94	1	2641.94	268.57	.00
ACADA_PRETEST	ACADA_POSTTEST	22657.56	1	22657.56	186.76	.00
	AFFECT_POSTTEST	1.79	1	1.79	0.18	.69
AFFECT_PRETEST	ACADA_POSTTEST	29.13	1	29.13	0.24	.62
	AFFECT_POSTTEST	710.67	1	710.67	72.24	.00
GENDER	ACADA_POSTTEST	2918.19	2	2918.19	24.05	.00
	AFFECT_POSTTEST	112.49	2	112.49	11.44	.00
GROUP	ACADA_POSTTEST	61794.45	1	61794.45	509.34	.00
	AFFECT_POSTTEST	43206.17	1	43206.17	4392.18	.00
GENDER*GROUP	ACADA_POSTTEST	4148.19	1	4148.19	34.19	.00

	AFFECT_POSTTEST	1093.33	1	1093.33	111.14	.00
Error	ACADA_POSTTEST	46102.30	382	89.04		
	AFFECT_POSTTEST	3738.09	382	50.62		
Total	ACADA_POSTTEST	1238272.00	386			
	AFFECT_POSTTEST	1427762.00	386			
Corrected Total	ACADA_POSTTEST	128623.25	385			
	AFFECT_POSTTEST	59916.19	385			

a. R Squared = .642 (Adjusted R Squared = .638) b. R Squared = .938 (Adjusted R Squared = .937)

The overall F test for the MANCOVA revealed significant group differences in the mean for the male and female in experimental groups and control group for the two dependent variables: academic achievement, $F(1, 380) = 13.07$, $p < .00$; affective outcome, $F(1, 380) = 53.41$, $p < .00$. The multivariate component of the MANCOVA revealed a significant effect of academic achievement, Wilks' Lambda $F(5, 380) = 17.79$, $p < .00$. That is, controlling for the covariates in the model, the pre achievement and affective outcome remained a significant predictor of scores on the dependent variables for the male and female in experimental and control groups. This result indicates that there is a significant difference among male and female students and their overall academic achievement and affective outcome in the learning of Mathematics. This difference was in favour of female students of the experimental group.

➤ Qualitative Findings

In addition to the collection of quantitative data, this study incorporated a robust qualitative component, aimed at deepening the understanding of students' learning processes and academic achievement in Mathematics. The qualitative data were collected and analyzed using a methodological approach known as template analysis. This analytic technique provides a structured framework that enables the researcher to systematically examine and interpret participants' responses. It involves both inductive and deductive coding procedures inductive in the sense of identifying emerging themes from the data, and deductive in the application of pre-defined codes based on the study's conceptual framework or theoretical orientation.

Template analysis is particularly effective in studies that seek to explore how individuals make sense of their learning experiences, as it offers flexibility in code development while maintaining analytical rigor. In the context of this study, template analysis served as a valuable tool for making sense of students' written responses, allowing for the identification of recurring themes that reflect varying levels of understanding and problem-solving ability in Mathematics.

The discussion of the qualitative findings is grounded in narrative illustrations, with selected excerpts from students' written work used to illuminate the nature of their problem-solving approaches. These narrative examples serve as windows into the students' cognitive and metacognitive strategies, offering rich, contextualized insights that complement the numerical trends identified in the quantitative analysis. By closely examining these written samples, the researchers were able to evaluate how students articulated their mathematical thinking and applied conceptual knowledge to solve problems.

To effectively demonstrate how students' writings reflect their problem-solving processes and overall academic achievement, the researchers developed a detailed rubric comprising five key dimensions of mathematical performance. These include: (1) the development of problem-solving skills, (2) the enhancement of conceptual understanding, (3) the demonstration of procedural fluency and application, (4) the use of mathematical reasoning, and (5) the development of content connections. Each of these components represents a critical aspect of mathematical proficiency and provides a lens through which student progress can be assessed.

For illustrative purposes, the study presents sample student responses corresponding to varying levels of performance as determined by the rubric. These samples serve to contextualize and exemplify how students perform across the spectrum of achievement, from developing to proficient to advanced levels. For instance, a student whose work demonstrates strong procedural fluency but weak conceptual connections may still achieve partial success, highlighting areas for targeted instructional support.

To ensure clarity and coherence in presenting the qualitative findings, the research sub-questions were used as guiding structures for organizing the discussion. These sub-questions provided a framework for interpreting how students' written responses aligned with the instructional goals of the intervention and how those responses reflected measurable learning outcomes. This approach not only aids in reader comprehension but also reinforces the connection between research objectives, data analysis, and the interpretation of findings. Ultimately, the integration of template analysis with rubric-based evaluation of student writing offers a comprehensive view of how writing-to-learn strategies reveal students' mathematical thinking. It also emphasizes the value of qualitative inquiry in educational research by uncovering dimensions of learning that may not be captured through traditional assessments alone.

➤ Sub-Research Question

What describes the development of problem-solving skill in student's writings as they solve problems in mathematics?

To answer this question, the researcher first looked at examples of student work for each score using a rubric. For Writing Prompt 10, see Appendix H, page 309, students were asked to construct a pie chart from a given data. Using the rubric, a student scored a four for the academic achievement of development of problem-solving skill if the student identified the goal of the problem or task, developed a plan that showed an understanding of all components of the problem, and executed the plan with no errors. A student scored a four for development of problem solving skill as they solved this problem.

- *She Wrote:*

✓ *To Construct the Pie Chart Multiply the Amount by the Number of Lecturer in Each of the Steps to Have;*

- ₦1,050,000
- ₦2, 744,000
- ₦4,830,000
- ₦3,780,000
- ₦2,090,000

Total number of staff is 400, divide each of the values in each steps by 400 and multiply by 360 correct to the nearest degree to have 26° , 68° , 120° , 94° and 52° . Use this to construct the pie chart.

✓ *The Mean Salary in the University is $\text{₦ } 14,494,000/400 = \text{₦}36, 235$.*

✓ *The Mean Contribution will be $\text{₦}36,235 \times 0.5 + \text{₦ } 500 = \text{₦}2,311.75$.*

She immediately started the process of constructing a pie chart. She identified the total number of staff and their steps. She calculated the appropriate degrees that will represent the sectors for each steps. She calculated the correct mean salary and mean contribution, and constructed the pie chart correctly. She chose to describe how she solved the problem after she had completed the problem. A score of three on the rubric for academic achievement in the development of problem-solving skills was assigned to a student who successfully identified the objective of the task and demonstrated a plan indicating an understanding of the problem, albeit with minor execution errors. One student who earned this score provided the following response:

Construction of the pie chart requires that you multiply the amount by the number of lecturer in each of the steps to have the degree that will form each sector of the circle. Also you should have idea of how to calculate sector of a circle.

Divide each of the values in each steps by 400 which is the total number of staff and multiply by 360 total degree that makes up a circle correct to the nearest degree to have 26° , 68° , 120° , 94° and 52° . Use this to construct the pie chart. (b) The mean salary in the university is $\text{₦ } 14,494,000/400 = \text{₦}35, 235$. (c) The mean contribution will be $\text{₦}36,235 + \text{₦ } 500 = \text{₦}36735.00$.

The student accurately identified the number of staff and their respective categories. The calculation of degrees for each salary category was also correctly performed, and the pie chart was properly constructed. However, the student made an error in calculating the mean salary and contribution, neglecting to multiply the total salary by 5%, which caused the mistake. The researcher noted this and asked the student about it when papers were returned. The student explained that they had forgotten the 5% salary calculation. The error was detected because the student provided a written explanation of the steps, allowing deeper insight into their reasoning.

In contrast, a student scored a two for problem-solving skill development if they recognized the task's objective but misunderstood one or more aspects of the problem, indicating minimal comprehension. One such student correctly identified the total number of staff but failed to describe the steps for determining the degree of each sector of the pie chart, which was central to the task. While she identified the correct staff number, she miscalculated the angles representing the salary categories.

- *She Wrote:*

Divide each value by 400, which is the total staff, and multiply by 90, the number of degrees in a circle sector.

Though she showed some familiarity with the task, her method and execution were flawed. She exhibited partial understanding but failed to construct a correct plan.

A student who failed to recognize the problem's goal yet showed vague awareness of its nature and provided no coherent plan scored a one. A score of zero was given to responses that lacked any indication of understanding or attempt at problem resolution. However, no students in the study scored one or zero in the development of problem-solving skills. Out of 189 student writing samples, 76.3% received a score of four, 17% were assigned a score of three, and 6.7% received a score of two.

➤ *Sub-Question 2*

What describe the conceptual understanding in student's writings as they solve problems in Mathematics?

To explore this question, student responses were analyzed using a rubric. Writing Prompt 1 (see Appendix H, p. 309) asked students to make a recommendation regarding which photocopy machine (A or B) and repair contract a department should purchase, given that machines are replaced every three years. The statistical concept involved was calculating the expected value of a discrete random variable. A score of four for conceptual understanding was awarded to students who clearly identified key concepts and supported their understanding with examples or illustrations. She calculated and identified the cost of Machine A as ₦11,800 and the cost of machine B as ₦11,010. Her solution read:

*$E(X) = \mu x = 0(.5) + 1(.25) + 2(.15) + 3(.1) = 0.85 \text{ repairs per year. } 0.85 * 3 = 2.55 \text{ repairs} * \text{₦}200 = \text{₦}510. 10,000 + 510 = \text{₦}11,010. \text{ I would recommend buying Machine B because over the three years that the machine will be in operation, it will be cheaper. The expected number of repairs over the next three years is 2.55. Therefore, with this number of repairs, the cost of Machine B will be ₦11,010, and machine A will be ₦11,800. So, the purchase of Machine B will be a better idea.}$*

I recommend Machine B because it will be less expensive over three years. The expected number of repairs is 2.55, leading to a total cost of ₦11,010 for Machine B and ₦11,800 for Machine A. Thus, purchasing Machine B is more cost-effective.

The student used appropriate notation and terminology (e.g., "expected number of repairs") and applied statistical concepts correctly, demonstrating a deep understanding.

A score of three was assigned to students who addressed major concepts but may have overlooked minor details or failed to effectively link examples to mathematical concepts. One such student calculated costs similarly but stated:

I recommend buying Machine B because Machine A's total is ₦11,800, which includes base price and a flat contract. Machine B's expected cost is ₦11,010. I calculated this using probability: $10,500 + 200(3(0.5 + 1*.25 + 2*.15 + 3*.1))$.*

Although he employed the correct method, he did not fully explain the "expected number of repairs" or elaborate on the conceptual application, reducing the clarity and depth of understanding. A score of two was given when students identified major concepts but had flawed reasoning or ignored essential details. For example, one student identified Machine A's cost as ₦10,600 and Machine B's as ₦11,100 and wrote, " $E(x) = 85$," suggesting awareness of expected value but with incorrect application. She added:

I would recommend buying machine B because the total cost for Machine A is ₦11,800. This is the base price plus the flat repair contract for three years. Machine B's total expected cost is ₦11,010. This is the base price plus the expected cost of repair based on a plan that charges per repair. I found this by calculating probability. $10,500 + 200(3(0.5 + 1*.25 + 2*.15 + 3*.1))$.*

From his calculations, it is clear that he understood the primary statistical concept being used in this problem, but omitted a minor detail regarding expected value. He used the phrase "expected cost." While his calculations demonstrated the process for finding the expected value, he did not discuss the expected number of repairs and how he applied that to the cost per repair. He indicated that he found the expected cost by calculating the probability. This was vague and did not completely convince the reader that he understood the statistical concept completely. By having students write in Mathematics, it enables a teacher to better grasp the conceptual understanding for students of topics being learned.

Using the rubric designed by the researcher, a student scored a two for the academic achievement of conceptual understanding if the student identified and provided support for major concepts but may have had minor errors in logic or understanding, and minor details were ignored or supported with incorrect or flawed thinking. A student scored a two for conceptual understanding as she solved this problem. She identified the cost for Machine A as ₦10,600 and the cost of Machine B as ₦11,100. She noted, " $E(x) = 85$." This notation indicated that she knew the primary concept that needed to be used to solve the problem. Yet, she made errors trying to apply it to the problem. She wrote:

I would suggest that Machine B be purchased. For one year, with no repairs, Machine B is ₦100 cheaper than Machine A. There is a 0.5 probability that Machine B will not need any repairs, while one repair has a probability of only 0.25. Machine B would save more money.

When analyzing her written explanation, it is clear that she chose to go away from the primary concept of expected value since she did not use the concept when drawing a conclusion about which machine to choose. Using the rubric, a student will scored a one for the academic achievement of increasing conceptual understanding if the student did not correctly identify major concepts and the information contained errors in logic or understanding. There were no scores of one for the example involving expected value. To analyze a written sample submitted by a student that received a score of one, we will look at a different problem. For Writing Prompt #8, see Appendix H, students were faced with a problem regarding the job that pays better. The primary concepts involved in this problem included conditional variation in the two jobs. A student scored a one for the academic achievement of increasing conceptual understanding as he solved this problem. He wrote:

For the initial solution, I calculated the weekly pay for both Tantalizer's and Tilapia's. I got ₦15,000 and ₦17,000, respectively. To answer the question, I checked it this way "Tantalizers pays you ₦900 an hour + the cost of uniforms. Tilapia pays you ₦850 an hour, no uniform. Tilapia pays more." When asked about that question during the focus group, he replied, "most work places require you to buy at least five uniforms in the beginning so you have some for the week and you are not coming to work smelly, like if it's a required uniform, they usually have you buy at least five at a time, not just one."

If he had added that explanation on the paper he turned in, he would have gotten more points per the rubric. It is evident that he did not know the major concepts involved with the problem. He described a significance test involving proportions while his actual work to solve different aspects of this problem did not include the conceptual understanding of the problem. When trying to solve a problem, it is essential for students to understand the primary concepts of a problem in order to develop and execute a strategy that will yield the solution. No study participants scored a zero for conceptual understanding. In the 189 writing samples collected for the study, 57.8% of the samples were coded a four, 23.7% of the writing samples collected for the study, were coded a three, 13.7% of the samples coded a two and 4.9% of the writing samples collected for the study, were coded a one for conceptual understanding.

➤ Sub-Question 3

What describe the demonstration of procedural understanding in student's writings as they solve problems in Mathematics?

To answer this question, students work for each score will be examined. For Writing Prompt 3, see Appendix H, students encountered a problem that there are 14 boys and 16 girls in Mrs Uko's class. What ratio best represent the relationship between the number of boys and the total number of students in Mrs Uko's class. (a) 7/15 (b) 7:10 (c) 7/9 (d) 14:16 (e) 8:15. They were to give explanation to their choice and analyze the question.

Using the researcher designed rubric, a student scored a four for the academic achievement of demonstration of procedural understanding if the student selected and executed appropriate strategies, and the representations and algorithms were appropriate. A student scored a four for demonstration of procedural understanding as she solved this problem. She identified the correct procedure of the problem.

The correct answer is A. It's critical to read this question carefully and it's easy to make a careless mistake on this one. The question gives the number of boys (14) and the number of girls (16), but it asking for the ratio of boys and the total number of students (14+ 16=30). That was sneaky. The ratio can be written as fraction 14/30 or 14: 30, but having a second look none of this is in the options given. Therefore, I need to reduce this to its lowest term by dividing through by 2 to have 7/15 which could also be written as 7:15 the correct option. I observed also that option for the ratio of boys to girls is given (option C and D), and the (Option E). The question is not a tough one conceptually, but it's an easy one to miss if you are not paying enough attention.

She followed a step by step procedure in solving the problem. This further supports that the student selected the correct procedure and used appropriate algorithms.

Using the designed rubric, a student scored a three for the problem-solving process of procedural understanding if the student selected and executed appropriate strategies, and some representations and algorithms had minor errors but did not affect the solution. For Writing Prompt 3, see Appendix H, a student scored a three for the demonstration of procedural understanding as she solved this problem. She wrote the number of boys, girls and total number in the class correctly, identified the appropriate steps to follow, computed the ratio with no problems, and wrote a reasonable conclusion.

She wrote: To find the ratio of boys to the total number in class, I need to have the total number in the class. I then expressed it as a fraction and later as a ratio. None of the options had the correct answer, then I tried reducing the fraction to its lowest term and this gave the correct option A.

She described the procedure that she took as she solved the problem. However, she missed some important procedures during her process of solving the problem. She did not discuss how she got the total number in class in step one. This is an important procedural step that she bypassed.

Using the rubric, a student scored a two for the academic achievement of demonstration of procedural understanding if the student selected an appropriate approach, but the execution was flawed. Also, representations and algorithms were appropriate for the task but were not executed properly. For Writing Prompt 1, see Appendix H, students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, A or B, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable. A student scored a two for demonstration of procedural understanding as he solved this problem.

He wrote: $(0)(.5) + (1)(.25) + (2)(.15) + (3)(.1) = .85$. *Since this problem involves finding the mean of a discrete random variable, I used the formula mean = $x_1 p_1 + x_2 p_2 + \dots$ to calculate the expected value of the repairs that machine B will amass over a 1 year period. With machine A, the department supervisor would have to pay ₦10,600 a year after purchasing the repair contract. Although machine B is more expensive initially, there is no repair contract. It is ₦200 for each repair. According to the chart, there is an expected value of .85 repairs per year. This indicates that there is a fairly strong probability that no repairs will be necessary. Therefore, I would recommend machine B.*

He understood that the primary statistical concept of this problem involved expected value. While he chose the correct path to solve the problem, his execution was flawed. After finding the expected value, he recommended Machine B. He incorrectly reasons that there is strong probability that no repairs will be needed. When she looked at the aspects of costs, he mentioned the costs for Machine A after the first year. He failed to apply the concept of expected value to the context of this problem. His lack of conceptual understanding seemed to affect his procedural understanding. He failed to mention anything regarding a three-year plan for each machine. He did not correctly compute three-year totals regarding each machine. No one that participated in the study scored a zero for the demonstration of procedural understanding.

Using the rubric, a student scored a one for the problem-solving process of procedural understanding if the student selected an inappropriate approach or selected the appropriate approach but could not begin implementation. Also, the representations and algorithms were not appropriate for the task. A student scored a one for procedural understanding as she solved this problem. She correctly identified the concept of the problem. It was unclear where one step ended and the next step began with her work. This may be due to her lack of understanding regarding this problem, which further supports her lack of procedural understanding for this question. In the 189 writing samples collected for the study, 79.3% of the samples were coded a four, 6.7% of the samples were coded a three, 9.7% of the samples were coded a two and 4.3% of the samples were coded a one for procedural understanding.

➤ Sub-Question 4

What describe the development of mathematical content connections in student's writings as they solve problems in Mathematics?

To answer this question, we will first look at examples of student work for each score. For Writing Prompt 9, see Appendix H, the second most common pieces of writing submitted which showed growth in the student's mathematical content connection was writing that summarized several lessons. The students who submitted a writing sample included a summary paragraph of a unit or chapter. For instance, one writing prompt included, asked the students to explain each of the three ways to solve systems of equations (see Appendix H). In response to this prompt the student explained each of the three methods and then also states which method was their favourite and why. The students then solved one system of equation using their favourite method. Using a rubric designed by Pugalee (2005), a student scored a four for the student's development of Mathematical content connection as they completed a problem, the researcher and panel of experts looked for evidence that the Mathematics were accurate, all Mathematical concepts and ideas were accurately identified, and Mathematical terms were used appropriately and connections were made to real life situation. A student scored a four for development of mathematical content as she solved this problem. She identified the formula for each of the methods. She used the correct variables in the correct places and concluded the correct solution.

She wrote: *For graphing, you get the y on its own. Then plot the graph using the y-intercept and the slope. The solution to the system of equations is where the two lines intersect each other. If the lines do not intersect, they are parallel. That means there is no solution to the system of equations. In substitution, you get one of the variables by itself. Then plug the expression into the other equation and solving for one variable. Last find the other variable by plugging in the number for the variable you solved. In elimination, you have to get the equation in standard form. Get the coefficient of one variable to be the same by multiplying it (them). Add if the signs are the same, subtract if they are different. Once you have found one variable, then plug it in and find the other. Elimination is my favourite because I found it the easiest. Graphing is confusing because of the slope and substitution is difficult because I have trouble getting the variables by themselves.*

Her mathematics was accurate. She substituted the given data in the correct places, enabling her to reach the appropriate solution.

For Writing Prompt 8, see Appendix H, the second example of writing students submitted that fell into this category dealt with graphing quadratic functions. In the prompt students were asked to compare two quadratic equations by writing generalizations to a friend. Some of the students addressed each of the points mentioned, writing paragraphs to explain each of the terms in their own words. Using a rubric designed by the researcher, a student scored a three for development of mathematical content connection if the mathematics were accurate, mathematical concepts and ideas were accurately identified, and mathematical terms were used appropriately and connected to real life situation, but there were minor errors. A student scored a three for development of mathematical content as she solved this problem. She wrote:

The role of "a" is that if "a" is positive the vertex will be the minimum of the graph and the graph will open upward. The minimum is the lowest point on the graph. The vertex is where the graph changes direction. If "a" is negative, the vertex will be the

maximum of the graph and the graph will open downward. "C" is the y-intercept. It is where the graph crosses the y-axis. In each of these cases, "c" is also the vertex.

Many students also included a well-labeled graph to go with this explanation. Each of the parts mentioned above would be pointed out in the graph. So the writing activity seemed to reinforce other mathematics work by the student. She identified the correct mathematical concepts that were needed to solve this problem. Her steps as she calculated the solution was flawless, except for one minor error. She did not identify each of the variables on the graph. This was likely a careless error. The researcher circled this on her paper without an explanation. When she received her paper back the next day, she explained it to be an oversight. She said, "I missed that." She was frustrated, but it was clear that she understood the mathematical content of the problem.

Using the rubric, a student scored a two for the problem-solving process of mathematical content if the mathematics contained minor errors, mathematical concepts and ideas were identified but with minor errors, and there were notable errors in the use of mathematical terms. No student scored a two for development of mathematical content as they solved the problem. Using the rubric, a student scored a one for development of mathematical content connection if the mathematics was mostly inaccurate, mathematical concepts and ideas are identified with several errors, and mathematical terms are used inappropriately. A student scored a one for problem-solving process of mathematical content as she solved this problem. The first part of the writing prompt asked students which die should be chosen to win the game. She wrote:

The role of "a" is that it shows the minimum and maximum point on the graph. The point is vertex "C" is the y-intercept. It is where the graph crosses the y-axis.

A correct way to have approached this problem included drawing a graph, identifying each of the variables on the graph and explaining what makes the vertex to be at maximum or minimum point on the graph. This process would have led her to the appropriate solution to the problem. She failed to mention any of these. The mathematical content used in her solution contained many errors. No one that participated in the study scored a zero for development of mathematical content. Going by the 189 writing samples collected for the study, 76.9% of the samples were coded a four, 17.2% of the samples were coded a three and 5.8% of the samples were coded a one for development mathematical content connection.

➤ Sub-Question 5

What describe the mathematical reasoning in student's writings as they solve problems in a statistics course?

To answer this question, we will first look at examples of student work for each score. For Writing Prompt 1, see Appendix H, students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, A or B, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable. Using the rubric, a student scored a four for the student's demonstration of Mathematical reasoning as they completed a problem, the researcher and panel of experts looked for evidence that the participant completely and accurately provided justification for major steps or processes, and defended the reasonableness of the answer with supporting reasons. A student scored a four for demonstration of mathematical reasoning as he solved this problem. After making a probability distribution table, he wrote:

$E(X) = 0(.5) + 1(.25) + 2(.15) + 3(.1) = .85 = \text{expected number of repairs per year}$ $(3.85) \times 200 = \text{¥}510 \text{ in three years} + \text{¥}10,500$
 Machine A: ¥11,800 in 3 years Machine B: ¥11,010 in 3 years I began by addressing machine A, and I found out that with a monthly repair cost of ¥50 for three years, that it would cost ¥1800. When added to the cost for the machine, it would cost ¥11,800. I found the expected amount of times the machine would need repairs in a year for 1 year, and then found how many in three years, and then costing ¥200 a repair. The repairs in three years on Machine B would be ¥510. When the repair cost is added to the cost of Machine B, it would cost ¥11,080. Therefore, we would choose Machine B.

She made no mistakes while solving this problem. It is obvious from her explanation that she understood the basis of this problem as she provided key justifications for each step. Notice how she stated that she found the "expected amount of times the machine would need repairs" in her explanation. This statement made it clear that she understood the statistical concept in the context of the situation. She leads the researcher through her journey as she solved the problem and why she took each particular step. She left no doubt to the researcher that her mathematical reasoning was sound.

For Writing Prompt 20, see Appendix H, students were asked to find the value of . Explain the steps used in solving the problem to support their answer and Identify the Mathematical concept(s) that they used to solve the problem and define each one. Using the rubric, a student scored a three for the academic achievement of demonstration of mathematical reasoning if the student accurately provided justification for major steps or processes but lacked clarity or detail, and defended the reasonableness of the answer but had minor omissions or errors in describing the approach. A student scored a three for problem-solving process of mathematical reasoning as he solved this problem. He wrote:

We must assume that, Completing the square on the first equation yields $(-3)^2=9$ -. If this equation is equivalent to $(-)^2=7$, then if $=3$ then $=2$. Completing the square on the second equation yields $(-3)^2=11$ -. Since this equation is equivalent to $(-)^2=$, we can see that $=3$ so it follows that $=2$ and $=9$.

This was correct and enabled the researcher to see that he understood one of the big concepts of the problem but lacked clarity and justification of the reasonableness of the steps taken to solve the problem. A writing sample from a student for the previous prompt, Writing Prompt 20, will be used to discuss and analyze a score of two. Using the rubric, a student scored a two for the problem-solving process of mathematical reasoning if the student provided justification for most of the steps or processes with no errors, defended the reasonableness of the answer, but may not have developed supporting reasons for the answer. A student scored a two for problem-solving process of mathematical reasoning as she solved this problem.

She wrote: *The formula for finding area of the rectangle will be stated first before proceeding with the problem. I remember that $(-1)^3 = (-1)(-1)(-1)$, which is as $3-3$.*

On the student's ASW worksheets, she wrote down the area formula before proceeding with the problem. The student "remembered" that $(-1)^3 = (-1)(-1)(-1)$, but failed to "understand" how to find the product when she incorrectly simplified it as $3-3$ in both of her solutions. This student "remembered" proper exponent rules but did not "understand" the context enough to "apply" them and execute a solution. She did not provide justification as to how she determined the facts.

For Writing Prompt #10, see Appendix H, students were asked to say which job pays more. Using the rubric, a student scored a one for the problem-solving process of mathematical reasoning if the student provided some justification for steps or processes but the response contained numerous errors and limited or no supporting evidence defending reasonableness of answer. A student scored a one for problem-solving process of mathematical reasoning as she solved this problem.

She wrote: *My opinion is that Tantalizer pay more because, Tantalizers pay ₦72,000.00 and Tilapia pay ₦68,000.00 the amount paid by Tantalizer is more than that of Tilapia.*

It was apparent that she truly did not understand this problem through her writing. She provided justification for her steps; yet, her justification for steps was incorrect and contained many errors. No one that participated in the study scored a zero for the problem-solving process of mathematical reasoning. Reviewing the 189 writing samples collected for the study, 78.7% of the samples were coded a four, 7.3% of the samples were coded a three, 10.3% of the samples were coded a two and 3.6% of the samples were coded a one for mathematical reasoning.

➤ Sub-Question 6

How do the student's writings improve over time as they utilized "Flash back in the alternative solution worksheet ASW?"

This study also sought to investigate the extent that students report "flash back" strategies when completing Alternative Solution Worksheet (ASW) activities, and the relationship between the subjects' reported use of "flash back" strategies and their performance on ASW activities. Despite the fact that the participants were unable to utilize "flash back" strategies effectively at the beginning, several patterns emerged within the transcripts of "think alouds", focus groups, observation notes, and the student's writings. These patterns include examples of student "Flash back" within specified levels of Bloom's Revised Taxonomy (BRT), student reactions to the process, affective changes, and strong and weak uses of "flash back."

The six levels of BRT provided a lens that the researcher was able to use to analyze student's "flash back" techniques as related to their struggles. BRT helped in understanding how the students were using "flash back" and why they were largely effective. One particular example, from a student's "think aloud" illustrates the taxonomy at work.

Remembering; The lowest level of BRT is simply recalling information. The researcher observed the students doing this frequently throughout the process. During the "think aloud" process, one student repeated "area equals length times width" after he read the question as well as at other times. On student's ASW worksheets, many students wrote down the area formula before proceeding with the problem. The student "remembered" that $(-1)^3 = (-1)(-1)(-1)$, but failed to "understand" how to find the product when she incorrectly simplified it as $3-3$ in both of her solutions. In this work the student "remembered" proper exponent rules but did not "understand" the context enough to "apply" them and execute a solution.

Understanding. The next level of BRT is explaining or interpreting ideas or concepts. For example, after the "think aloud" in which one student recalled the formula for area of a rectangle in the understanding phase, he noted *that* ;

"Area of the rectangle equals $2-11-6$, and one of the sides is $5+2$, so I'm going to try $5+2$ in parenthesis multiplied by $2-11-6$, no we're dividing."

He even drew a figure that accurately represented the information. This student understands how each piece of the given information fits in the context of the problem. This shows that the student understands the relationship between area and length to realize he should divide. The student also recognizes that relationship when she notates $(5+2)(?) = 2-11-6$. In all the above instances, the students understand the relationship between the expressions given, and this helped her move on to the next level of BRT to get a solution.

Applying. The third level of BRT is using the given information to execute a solution. At this level, the researcher observed the students trying to do some mathematical operations to solve the problem. Some students were stuck at this point. During the “think aloud”, one student wavered between the applying level and the next level, analyzing. In the previous example, he realized he should be dividing, rather than multiplying. His process as follows.

So $(5+2) \div (2-11-6)$ that makes ...no... I will multiply it. [He goes back to multiplying even though he knows he should be dividing.] So $5+2$ multiplied by that, is 5^3-55^2-30 , then 2 times 2 which would be 2^2 , then 2 times -11 is -22 and then 2 times -6 is -12 . Like terms so it would be $5^3-107+2^2-12$, then that wouldn't work because it's supposed to be length times width equals area and the area is $2-11-6$, so we will do $5+2$...I don't know if I add, divide, multiply, or subtract, yeah I can try to subtract, that's what I will do. [Again, he realizes that he should not have multiplied. He thinks he must add, subtract, multiply, or divide, so he settles for subtraction next even though he already stated he has to divide.] So I can take $2-11-6$ and subtract $5+2$ from that and that gives me $2-16-4$. No, I got to find the area I can't factor it so I'm going to come back to this one later. [Once again, he knows what he did was wrong, but he cannot figure out how to make it right. He is using flash back here, but at the levels of understanding and applying.] Question 1 again. Area equals length times width. Let's see, we can't factor the $2-11-6$. We can't factor it, and we can't subtract it so how are we supposed to ...So we may have to divide, no we can't divide. So $2-11-6=5+2$ so we can subtract the 11 from both sides, then we can...this looks right. So $2=16+8$, then divide the 2 so it will be $=16+8^2$ and that's what the area is...no that's not what the area is...no, it's supposed to be area equals length times width and the $5+2$ has to be the length or the width and the $16+8$ has to be something. So my final answer will be $5+2$ equals the length and then $16+8$ equals the width and the area is $2-11-6$.

At this point in the “think aloud”, the student is trying to analyze his solution (the next level of BRT), but he stopped short of actually doing it.

Analyzing. The fourth level of BRT is breaking information into parts to explore understandings and relationships among the data. In mathematical problem solving, this level differs from the previous level, applying, in that students who are analyzing are pointedly pursuing a solution based on their understandings of the relationships in the problem rather than just randomly doing mathematical operations for the sake of doing something. In the example above, the student never really got to this level. He knew he was supposed to divide, as he stated multiple times, but since he did not figure out how to do the division, he settled for multiplying and subtracting. Evaluating. The fifth level of BRT is justifying a decision or course of action. In mathematical problem solving, evaluating can be accomplished by explaining why one chose a certain solution method and then checking the solution obtained to validate it. In the prior example, the student write up expressed this when she wrote;

$E(X) = 0(.5) + 1(.25) + 2(.15) + 3(.1) .85 =$ expected number of repairs per year $(3.85)*\text{N}200 = \text{N}510$ in three years + $\text{N}10,500$ Machine A: $\text{N}11,800$ in 3 years Machine B: $\text{N}11,010$ in 3 years I began by addressing machine A, and I found out that with a monthly repair cost of $\text{N}50$ for three years, that it would cost $\text{N}1800$. When added to the cost for the machine, it would costs $\text{N}11,800$. I found the expected amount of times the machine would need repairs in a year for 1 year, and then found how many in three years, and then costing $\text{N}200$ a repair. The repairs in three years on Machine B would be $\text{N}510$. When the repair cost is added to the cost of Machine B, it would costs $\text{N}11,080$. Therefore, we would choose Machine B.*

Here “flash back” is most important, to assess the validity of a solution.

Creating. The sixth and highest level of BRT is generating new ways of viewing things, or generating alternative solution methods, as was requested of the students during this study. Some students were largely unsuccessful at this level, but many of them were successful. For one example of a student problem posing, a written sample from Writing Prompt 1, see Appendix H, will be used. Students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, A or B, along with its repair contract, should be purchased. The primary statistical concept involved the process of calculating the expected value of a discrete random variable. Students were asked to pose a problem of a different context using the same statistical concept of the original written prompt. A student wrote:

A pet lover is considering taking his dog to two different veterinarian offices in town. Vet office A charges a one-year membership fee of $\text{N}250$, while office B charges a membership fee of $\text{N}300$. The pet lover plans to sell his dog in two years when he goes off to college. A vaccination contract costs $\text{N}10$ per month at office A and $\text{N}25$ per month at office B. The distribution of the number of shots per year is as follows:

Number of shorts	0	1	2	3	4
Probability	0.05	0.1	0.15	0.25	0.45

The pet lover asks you to suggest which of the two he should go with. Which would you choose and why?

After reading this student's problem involving the statistical concept of expected value, the researcher wondered why he had chosen a problem involving pets. When the response was returned, the researcher asked him and he stated, "I love dogs I have two dogs, and I spend a great deal of time reading about dogs." He took a statistical concept and posed a problem in a context of which he was familiar. It is clear that he understood the statistical concept of expected value when reading the problem that he posed. The process of problem posing assisted this student by helping cultivate his mathematical thinking and further develop his creativity and understanding. The result showed that majority of the students used strong flash back and were able to solve the problem successfully to get the correct answers, which reflected in their high scores in the achievement test. Few students used weak flash back and could not get the correct answer, which gave rise to their low scores in the achievement test.

➤ *Effect of Writing to Learn and Triangulation Assessment Tools and Methods on Academic Achievement and Affective Outcome of Students in Mathematics*

To fully address the fourth and fifth research questions, a template was created based on the questions students responded to in their initial writing excerpt for the course. In the first week, all participants completed a *Learning Biography* which required them to reflect on who they are as students, their general feelings about mathematics, their strengths in the subject, their goals for the course, long-term educational aspirations, and reasons they believed they could be successful. Students were asked specifically not to focus on the grades they aimed to earn, but rather on "who they are" as learners. The resulting template consisted of five constructs: self-concept, interest, attitudinal change, motivation, and value. These constructs guided the coding of all participants' responses.

Initial data extraction revealed a largely superficial level of reflection. Most pre-course Learning Biographies failed to address all questions in detail and lacked in-depth self-assessment. To preserve anonymity, pseudonyms were assigned to all student responses.

➤ *Initial Template*

Under the course goals construct, students' responses were typically brief, often focused on grades, and revealed limited meta-cognitive awareness regarding their self-concept as learners.

John stated, "My goal for this class is to get a B or better. Also to learn all that will apply to an engineering job which I plan to do at the completion of my NCE. I want to get as much knowledge as I need to do a job in the engineering field well." Regina noted, "I want to have an A or B in this course when all is over because I know I can do it. I have taken GSE and I hope I can still remember some of the concepts that I learned in year one to make this class a little simpler." Akpan shared, "Of course I would like to receive a good grade, but I have greater goals for this class. As I mentioned before, I enjoy learning new material. Unfortunately, I couldn't get my desired grade in year one, so I decided that I would take this class very seriously. One goal is to challenge myself to learn more in order to be successful in the career I would like to pursue."

In terms of the type of student construct, comments also lacked depth and specific insights into students' identities as learners.

Evelyn wrote, "I enjoy learning and being challenged in my education. I have a wide range of interests, finding almost any topic enjoyable. This, coupled with a large work ethic, has allowed me to succeed in my educational career so far." Oku stated, "As a learner I am very inquisitive. I enjoy learning new things, and if I don't get it at first I work at it until I do. My Grandpa always told me that a wise person tries to learn something new every day, and the person who thinks they know it all is an idiot." Christina reflected, "I was a hands-on learner. If I learn at my own pace, I found that the material sticks with me."

Post-course *Learning Biographies* and inventory writings revealed greater depth, particularly in how students connected course concepts and explored their learning processes more critically. Prompts were crafted to encourage students to delve deeper into their learning approaches and self-reflection.

Chinedu observed, "From these new realizations, I was able to understand the other quadratic functions and their graphs. Soon, my portfolio became more of a quick referencing tool than a visual aid. I hope that I will be able to use my knowledge of quadratic equations as I move on to a higher-level math."

Rose remarked, "I now realize how much quadratic equation is in our world because of writing the connections we all can make through the concepts of this course. Quadratic equation is everywhere, and by doing the research to find out these things it makes learning quadratic a little easier because we actually know we can use these concepts in the real world." Daniel wrote, "I do see the connection between writing and math just like anything you learn, the more you think about what you need to work on, the more effort you will put into actually doing it."

Based on thematic analysis of various student writings threaded discussions, three mathematical growth journal entries, and the end-of-semester reflective essay the refined template highlighted six key themes: Attitudinal changes as a learner, connections and writing, feelings about or interest in mathematics, self-concept, motivation and value of writing, and changes as a learner.

The connections and writing construct uncovered multiple ways students linked mathematics to writing. They articulated conceptual connections, expressed deeper understanding through writing, and reflected on connections between mathematics, future professions, and real-life scenarios. The feelings about mathematics construct revealed strong attitudes either highly positive or negative towards the subject, with few reporting neutral or mixed feelings.

The reflection centered on the quadratic equation, a foundational concept in the course. Students' writings demonstrated their varying levels of reflection, either showcasing deep understanding through detailed connections or indicating surface-level understanding with minimal conceptual linkage. Students who consistently expressed meaningful connections offered richer, more detailed insights. For instance:

Evelyn reflected, "Over the past few weeks of basic general mathematics, the quadratic equation has transitioned from paperwork to an applicable tool. When this semester began, I had only heard mention of quadratic equation and quite frankly, they made absolutely no sense. Therefore, when we carried out the project on quadratic equation and real-life problems it can be used to solve, I was completely bewildered. Then we were instructed to use the concept of parabola that is, the graph of quadratic equation to find things around us that are having that curve, and the light turned on. As I worked through problems such as this and found the value of it, I realized the purpose of learning mathematics. The equation contains everything necessary for finding the solutions to these types of problems: the variation, the simultaneous equations, statistics, and many other problems even in other courses. It also provides a visualization of the relationship among the other science subjects. The continuation of the work with the quadratic equation has led to the discovery of more and more patterns and a development in my understanding of mathematics as a whole. As a result of the project, my grasp on this aspect of mathematics has increased tremendously."

Belema commented, "The parabola has followed me everywhere I go, and I am still in astonishment of how it is still applicable through all these objects. When we first talked about the graph of quadratic equations, I thought it was just something that would only be used for a certain one or two things. I also thought that I knew the quadratic equation well enough, but when the first test came around, I was not as familiar with it as I had thought. I got really angry at myself and was determined to do better on the next test. I did do better, but I still messed up by switching some values around. I would say that these tests helped me to get to know quadratic equations and indeed mathematics better because they spurred me on to do better. Going over all the connections that the equation had with what we were doing was very instrumental in my understanding of mathematics and how it all goes together. I was really amazed at how the concept of parabola could be used for graphing wave-like data, represent complex numbers, and do plotting in a polar-coordinate system. I see parabola on everything around me. I still find it crazy that all these topics rely on and are based from a quadratic equation. In the end, our constant going over the project and its applications helped me get a grasp on where mathematics really comes from."

Within the value of writing construct and based on students' comments, the researchers were able to determine if students saw any relevance in writing as it relates to their ability to reflect on what they had learned both between class sessions and throughout the semester. The intent of this study was to use writing as a tool to encourage students to engage in deeper reflection on connections across course concepts, and to help them realize the importance of self-concept in their learning processes. Through these written reflections, the study revealed aspects of students' individual metacognitive functioning and how it may have evolved over the course of the study.

Emen wrote, "Writing essays in a math class was definitely something new to me, but I think it was effective for various reasons. Writing out what was and what was not effective for me caused me to have a level of self-reflection that I had never really considered in other math courses. Because of the essays, I was encouraged to pinpoint what was helping me significantly, like my revelation about talking through concepts with others." Kelechi stated, "I agree that writing in mathematics helps ground the basics and proves what you really know in maths. I personally know that it proved that I really didn't know as much as I thought I did, but that is a good thing because now I know what to fix with myself. It is one thing to do something that makes sense in your head or to repeat some steps someone gave you, but to actually be able to quantify the data and put it down on paper is a whole different thing. This requires an innate understanding of whatever you are trying to write about in order to put it in a coherent and understandable way." Judith commented, "I totally agree that writing required me to reflect on what I really know and understand. Outside of not knowing the correct vocabulary, if I couldn't explain a step or process, then that told me I didn't understand it and needed to review. I guess it's kind of like 'the devil's in the details.'" Jack stated, "Writing to reflect on what I have learned has shown me how to really appreciate what I have been learning. It has helped me to see how beneficial everything I have learned is because I reflect on everything and it forces me to take a second look at it all." Alex said, "Yes, I suppose writing and explaining in your own words does make you look back and think about how much you really know on the subject. Also by doing this process it may help to learn a concept because you are forced to think longer and harder about the concept and see other connections to the concept."

A. Major Findings

Based on the analysis of the result presented in this chapter, the following major findings were made:

- There exists significant difference between the achievement scores of students assessed with writing to learn method and those assessed with the traditional method in favour of the writing to learn method. A significant difference was also observed for the affective outcome scores between the writing to learn method and traditional method in favour of writing to learn method.
- There exists significant difference between the achievement scores of students assessed with triangulation method and those assessed with the traditional method in favour of the triangulation method. A significant difference was also observed for the affective outcome scores between the triangulation method and traditional method in favour of triangulation method.
- There was a statistically significant mean difference between experimental and control group in respect to post-test scores of academic achievement and affective outcome. This means that writing to learn mathematics and triangulation of assessment method usage increased student academic achievement and affective outcome in the learning of mathematics positively.
- The nature of Self-concept, interest, attitudinal change, motivation and value in mathematics learning were significant predictor of academic achievement. Also, self-concept, motivation, value and interest contributed more significantly, attitudinal change contributed less significantly to the increase in achievement of students in mathematics.
- There was a substantive, high level and deeper reflective nature of meta-cognitive functioning change as students engaged in the intervention.
- There exists a significant difference between reciprocity method and project-based method in favour of reciprocity. The reciprocity methods contributed more to the increase in academic achievement of the students in mathematics while project-based contributed less. The two assessment methods are equally effective in increasing affective outcome of students in the learning of mathematics.
- Significant difference exists between writing to demonstrate knowledge and performance-based tools in favour of writing to demonstrate knowledge. However, writing to demonstrate knowledge tools contributed more to the increase in academic achievement of the students in mathematics while project-based contributed less. The two assessment methods are equally effective in increasing affective outcome of students in the learning of mathematics.
- There was no significant interaction effect of writing to learn as an assessment tool and the triangulation assessment techniques as method of assessing academic achievement, self-concept and interest of students in the learning of Mathematics. But these two methods of assessment had significant effect on academic achievement and affective outcome of students.
- There was a significant mean difference in male and female student's overall academic achievements and affective outcomes between the experimental and the control groups in Mathematics. This difference was in favour of female students.
- The writing samples of students who tended to do well when providing their solutions to problems consistently scored threes and fours using the rubric. These students were successfully able to demonstrate the five problem-solving processes as they solved problems, though students had lower scores in conceptual understanding when compared to other processes. However, not all students were successful in this manner.
- Majority of the students used strong flash back and were able to solve the problem successfully to get the correct answers, which reflected in their high scores in the achievement test. Few students used weak flash back and could not get the correct answer, which gave rise to their low scores in the achievement test.
- Majority of the students did make significant changes to their approach to learning, and they were able to make deeper and meaningful conceptual connections. It also was apparent that writing in mathematics and about mathematics encouraged students to reflect on what they were learning, and allowed them to make more meaningful connections about the content and themselves as learners and they experienced substantive changes and growth in their meta-cognitive functioning as a result of the intervention.

B. Discussion of Findings

Based on the analysis of the result presented in this chapter, the following major findings were discussed.

➤ *Effect of Writing to Learn Assessment Tools on Academic Achievement and Affective Outcome of Students in Mathematics*

The findings revealed that there was a significant relationship between writing to learn Mathematics as an assessment tool and overall development of problem-solving skills, increasing conceptual understanding, demonstration of procedural application, demonstration of maths reasoning, development of content connections (academic achievement) and self-concept and interest (affective outcome) of students with respect to learning Mathematics. A significant difference exists between the achievement scores of students assessed with writing to learn method and those assessed with the traditional method in favour of the writing to learn method. A significant difference was also observed for the affective outcome scores between the writing to learn method and traditional method in favour of writing to learn method.

The implementation of writing to learn as alternative assessment tools increased student's academic achievement and affective outcome more than the traditional method of assessment. Writing can give students more time to think, allow for multiple representations, and provide opportunities for students (especially quieter students) to communicate with the teacher. When students are stuck on a problem, they can write out their thought process and may be able to see their error and solve the problem. Additionally, when students recognize their confusion, it is a step toward understanding. Writing in Mathematics can also be used to support gifted and talented students by pushing them further. This finding is in line with the finding of Craig (2016) who

investigated how writing explanatory paragraphs in Mathematics class influenced problem solving behavior in 39 calculus students of University of Cape Town, South Africa. The researcher concluded that writing about problem solving in Mathematics can cause a reflective imbalance where students realize their reasoning is wrong or incomplete; and this in-turn, leads to a reflective process, which can eventually propel the student to correct the learning deficiency.

The findings agrees with the findings of Fauziah, Mardiyana and Saputro (2018), which was to learn the process of using mini project assessments on statistics learning which is conducted by lecturers and to discuss specifically the use of mini projects to improving student's learning in the school of Surakarta. The result of data analysis shows that the average score of rubric of student mini projects result was 82 with 96% classical completeness. This study shows that the application of mini project assessment can improve student's Mathematics learning outcomes. Also the findings agrees with Baxter (2008) who studied Mathematics writing with lower achieving 7th graders and found that writing in Mathematics increases student's comprehension and performance and therefore recommended teaching writing in Mathematics thoughtfully and gradually. Students should be able to engage in meaningful Mathematics communication, as writing more with numbers will empower the misuse of data and statistics. This finding also agrees with the finding of Baxter, Woodward and Olson (2005). Baxter et al., studied journal writing in seventh grade Mathematics classes and found that writing in Mathematics can be used to support lower achieving middle school students because it helps students who do not typically participate in class discussions take on a more active role in their learning. The finding also agrees with the finding of Knox (2017) that writing in Mathematics can help these student's develop conceptual understandings and problem-solving skills. He further reported that lecturers can analyze student's responses to evaluate each stage of the problem solving process.

➤ *Effect of Triangulation Assessment Methods on Academic Achievement and Affective Outcome of Students in Mathematics*

The findings revealed that there was a significant mean difference between triangulation approach as assessment method and overall development of problem-solving skills, increasing conceptual understanding, demonstration of procedural application, demonstration of maths reasoning, development of content connections (academic achievement) and (affective outcome) of students with respect to learning Mathematics. Triangulation acknowledges these limitations but presumes that when multiple perspectives point to the same conclusions, the odds are that the conclusions are valid. Triangulation assessment methods make students to gain a more comprehensive perspective on the development of student mathematics skills. This finding is in line with the finding of Gbore (2013) who examined relative effectiveness of three evaluation techniques (triangulation) on academic performance of secondary school students in integrated science in Ondo State, Nigeria and reported a significant difference in the performance of students exposed to closed book, open book and open time techniques of evaluation of learning outcome in Integrated Science, pointing out that student's performance in Integrated Science was better in closed book than open book and open time techniques while open book was better than open time technique.

The triangulation method is used to validate repeated assessments in order to arrive at a meaningful conclusion of the best assessment method. Data collected from several sources are analysed and statistically validated using the triangulation method. Triangulation method of assessment aims to validate assessment techniques so as to strengthen the observations from assessment methods. This finding is also in line with the finding of Nelson (2010) whose purpose of the study was to illustrate the benefits of two different approaches to triangulation and the findings of the study showed that the process of taking a focused question, examining available data and then presenting the question and the data to faculty, administration, and their Board of Trustees has increased very significantly the engagement of their colleagues in assessment and on closer analysis, using triangulation method many of the low scoring seniors had very high GPAs and some were majors or minors in the social and natural sciences.

➤ *Effect of Writing to Learn and Triangulation Assessment Tools and Methods on Academic Achievement and Affective Outcome of Students in Mathematics*

The findings showed that there was a significant mean difference in student's overall academic achievements and affective outcomes between the alternative assessment (writing to learn mathematics and triangulation of assessment method) groups and the traditional assessment group in Mathematics. Students in the alternative assessment methods did significantly better than students in traditional assessment method. This was seen in increases in academic achievement and affective outcome of students as measured by mean achievement and affective outcome scores. Writing to learn mathematics and triangulation of assessment method usage increased student academic achievement and affective outcome in the learning of mathematics positively. Traditional paper and pencil test has been the dominant method of assessment practice in schools. This is because of the much emphasis on test results and the Nigerian educational system which is still driven by examinations. Hence much attention has been focused on the assessment and evaluation of cognitive variables at the end of formal instruction which is always done through the administration of an achievement test in the subject areas.

Writing to learn Mathematics assessment make students to demonstrate that they have mastered specific skills and competencies by performing an activity to reveal what they are capable of doing. It offers students the opportunity to apply their knowledge and skills from several areas to demonstrate that they are capable of reaching a learning target and come up with their own solution. This finding is supported by the finding of Knox (2017) who reported in his study that writing to learn Mathematics assessment helped students realized that solving a Mathematics problem is about the process as it is not always just about getting the right answer or memorizing disjointed facts. This finding is also in line with the finding of (Cai and Brook, 2006) who found

out in their study that with alternative assessment methods, students came up with alternate methods for solving a problem because alternative assessment methods made students understand the problem, devise a plan, carry out the plan, and flash back at the completed solution, by reconsidering and re-examining the result and the path that led to it. Similarly, the finding agrees with that of Lee (2009) as he found out in his study that the use of alternative assessment solutions encouraged students to flash back and went further to report that students need more guidance to effectively flash back than just having the answers; however they need to be taught to problem solve reflectively and to share their thinking and to listen to the thinking of others.

➤ *Effect of Alternative Assessment Tools and Methods on Affective Outcome of Students in Mathematics*

The findings indicated that there was a significant relationship between the nature of student's individual meta-cognitive functioning (self-concept, interest, attitudinal change, motivation and value), and increased academic achievement in the learning of Mathematics. Meta-cognition as a flexible cognitive strategy help students regulate and have knowledge of their own cognition through planning, monitoring and evaluation of their studies. Students meta-cognitive functioning changed after the intervention which caused significant relationship between the meta-cognitive functioning change (self-concept, interest, attitudinal change, motivation and value) and academic achievement in the learning of Mathematics. Self-concept, interest, attitudinal change, motivation and value of mathematics learning could significantly predict participants' overall academic achievement in mathematics.

These constructs focused on the nature of student's individual meta-cognitive functioning and allowed the researcher to examine how student's individual meta-cognitive functioning changed during the intervention. Meta-cognition is linked with increased success of students learning situations because meta-cognition involves employing appropriate information and strategies during the problem solving process. Meta-cognitive experiences are conscious experiences that are cognitive and affective. This findings collaborates with that of Cross (2009) as he reported in his study that meta-cognitive processes helped students achieved higher scores in Mathematics, promote higher-level thinking and it is considered one of the most important factors in student learning.

Developing meta-cognitive skills and discoursing in the classroom have been tied with students having a deeper understanding and increased Mathematics achievement. With meta-cognitive skills, students are able to actively monitor and consequently regulate and orchestrate academic processes in relation to their cognitive properties which they exhibit, usually with the guidance of some concrete goals or objectives. Similarly, this findings also agrees with the finding of Carr (2010) who found out in his study and reported that meta-cognitive skills influenced student's Mathematics ability and caused the changes that need to occur for students to progress in Mathematics.

The findings revealed that there was a significant relationship between the meta-cognitive functioning change (self-concept, interest, attitudinal change, motivation and value) and academic achievement in the learning of Mathematics. A great percent of the variance in academic achievement was attributed to the variance of the combination of the five independent variables (self-concept, interest, attitudinal change, motivation and value of mathematics learning). The correlation between the dependent variable (academic achievement) and the combination of the five independent variables (self-concept, interest, attitudinal change, motivation and value of mathematics learning) was statistically significant as a result of the importance of these constructs in student's performance.

This findings is supported by the finding of Pugalee (2001) who reported in his study that meta-cognition was linked with increased success in student's learning situations. Meta-cognitive strategies help students to appropriately apply knowledge of their individual variables, knowledge of task variables and knowledge of strategy variables. These strategies make them learn effectively, thereby improving their academic performance. Similarly, the finding also agrees with that of Pugalee (2004) as the author found out and reported in his study that students who had sufficient knowledge in addition to having awareness and control of that knowledge were able to have high scores and success in relation to cognitive performance. Knowledge about one's own thought processes, self-regulation and beliefs and intuitions are necessary in learning situations because they aid students to develop skills to make realistic assessments of what they can learn. This requires students to reflect on their thinking and examine the accuracy of their thinking as well as making sure they can determine what they know about a problem prior to attempting a solution.

This findings is also in line with that of Lucangeli and Cornoldi in Reilly (2007) as they found out that Mathematics learning requires different levels of meta-cognitive involvement. Some aspects of Mathematics become automated processes over time and require less meta-cognitive involvement while other tasks such as problem solving demand complex and flexible thought processes. They further reported that assessment and teaching of meta-cognitive skills in Mathematics courses was a valuable component in predicting Mathematics abilities.

➤ *Interaction Effect of Writing to Learn and Triangulation Assessment Tools and Methods on Academic Achievement and Affective Outcome of Students in Mathematics*

The findings indicated that there was a significant mean difference between the assessment approach reciprocity with peer and project-based techniques of evaluation and increase in development of problem-solving skills, conceptual understanding, demonstration of procedural application, demonstration of maths reasoning, development of content connections (academic

achievement) and self-concept and interest (affective outcome) of the students in the learning of Mathematics. A significant main effect was observed for assessment methods with respect to post-achievement and post- affective outcome. The reciprocity and project-based assessment methods remained a significant predictor of scores on the academic achievement and affective outcome of students. It is necessary to balance the assessment of learning outcomes of learners by assessing all the domains associated with behavioural changes instead of assessing the cognitive achievement in the learner alone.

Teachers should assess the affective outcome in learners as this will enable learners not only to acquire academic competencies but to be adequately equipped with knowledge, skills, attitudes, values, practical and psychosocial skills that would enable them live healthy and satisfying lives and derive the benefit of learning. Learning is associated with behavioural changes in the cognitive (mental processes), affective (attitudes and feelings) and psychomotor (coordination between brain and muscles) domains and all these should be assessed with appropriate assessment methods.

This findings is supported by the finding of Miller and Mitchell, in Riplinger (2008) because he reported that Mathematics was taught symbolically with traditional method which was used to arrive at the answer and stated further that this traditional method of teaching, coupled with traditional assessment methods of paper and pencil tests, led to the development of Mathematics anxiety in some students but that reciprocity with peer and project techniques of assessment made students to have high scores and become proficient in Mathematics. Unfortunately, students with high levels of Mathematics anxiety avoid Mathematics both in schooling and career choices; when they do take a Mathematics course, they receive lower grades even though they may be competent. The finding of this study is also in line with that of Baxter, Woodward and Olson (2005) as they found that there was a strong relationship between interest and achievement in Mathematics and that Mathematical proficiency includes a productive disposition. Therefore, a student's attitude toward Mathematics may be a more important factor than previously thought.

The findings indicated that there was a significant mean difference between the assessment tools such as writing to demonstrate knowledge and performance- based and increase in the development of problem-solving skills, conceptual understanding, demonstration of procedural application, demonstration of maths reasoning, development of content connections (academic achievement) and self-concept and interest (affective outcome) of the students in the learning of Mathematics. Writing to demonstrate knowledge and performance-based assessment tools of the writing to learn assessment methods remain a significant predictor of scores on academic achievement and affective outcome of students. It is therefore necessary that examiners should complement closed-ended test technique of assessing learning outcomes with alternative technique like writing to demonstrate knowledge and performance-based assessment to reduce cheating behaviour, anxiety and fear of failing examination among the students. This will encourage them to add value to whatever they are learning and will increase their interest to learn Mathematics.

The techniques should be used intermittently to keep the students busy at home towards making them to acquire mastery of the subject matter and to motivate them to learn. The implementation of these methods would lead to improvement in the reading culture of the students and subsequently great achievement in the learning of Mathematics and successful academic achievement as well as affective outcomes. Triangulation involves multiple entities, assessing the same outcomes, using different methodologies to validate the findings. This finding is in line with the finding of Moon, Brighton, Callahan and Robinson (2005) because they found out in their study that with performance assessment methods, students perceive the learning process as important and related to skills used in the real world. Also, it is about purposeful teaching in the direction of experiences that are useful, reasonable and real, and in which the students was active learners and creators of knowledge and skills that are structured around meaningful contexts.

Performance assessment tasks can take several forms like presentations, writing-based projects, group work, problem solving activities, debating, and to make choices about their learning. The finding of Arshin (2015) on the effect of performance assessment-Driven instructional also collaborates this finding as he reported that performance assessment provided opportunity for students to gain real and meaningful experiences for themselves, increased their interest, motivated as well as demonstrated high-level thinking skills. He further reported that Mathematics performance-based writing assessment is a form of meaningful measurement of student learning outcomes for the sphere of attitude, skill and knowledge in Mathematics. Student's involvement in performing the task is meaningful for their personal development as it helps them to construct attitude, skills and knowledge achieved through the completion of writing tasks which involve active and creative role of the students.

The findings showed that there was no significant interaction effect of writing to learn as an assessment tool and the triangulation assessment techniques as method of evaluating development of problem-solving skills, increasing conceptual understanding, demonstration of procedural application, demonstration of maths reasoning, development of content connections (academic achievement) and self-concept and interest (affective outcome) of the students in the learning of Mathematics. But these two methods of assessment had significant effect on academic achievement and affective outcome of students. This finding agrees with the finding of Gbore (2013) which showed that triangulation method of assessment like project-based method (Home work/assignment) permitted students to complete assignments under preferred conditions of noise, light, design, mobility and time of the day with improved student's achievement, attitude and conduct.

Similarly, the findings of Alonge (2004) is also in line with this finding as he reported that writing to learn and the triangulation assessment techniques reduce stress and rote learning and gave valid judgement about students obtained scores. Both writing to

learn and the triangulation assessment techniques move away from the attitude of rote learning which aims at merely preparing students for examinations but propels students understanding of the concepts of the school subjects which are taught to them. This leads to inaccurate conclusions regarding student progress and students learning. The finding also agrees with the finding of Wiggins and McTighe (2005) because they found out and reported that students assessed using different methods of assessments such as performance task (project-based method), knowledge and skill (reciprocity with peer method) and criterion-referenced assessment (open-ended method) were found to have consistent scores indicating that they really assimilated the concepts taught. The methods rely on using multiple data sources and approaches to support a finding by showing that independent measures of scores agree among themselves or, at least, do not contradict themselves.

➤ *Gender Effect on Academic Achievement and Affective Outcome of Students in Mathematics*

The findings, overall, revealed significant group differences in the mean for the male and female in experimental groups and control group for academic achievement and affective outcome in the learning of Mathematics. This difference was in favour of female students. The higher scores obtained by female students might be as a result of more attitudinal changes, connections and writing, feelings about or interest in mathematics, self-concept, motivation and value of writing in female learners compared to their male counterparts. The connections and writing uncovered multiple ways in which female students were able to find links between mathematics and writing. Female students made conceptual connections, found deeper understanding about mathematical concepts due to writing about them, and were able to see connections related to writing and their future professions or writing and real world scenarios.

The male students seemed careless about the whole experimental procedures which made them less interested in the teaching learning process thereby leading to their lower scores compared to the female students who were in the opposite direction. The connections made by the female students through their writing showed deeper level of reflection as a result of multiple connections demonstrated within their writing throughout the course and a surface level understanding demonstrated by their writing in which the male students presented only minimal connections between the concepts in the course. The findings is in agreement with the finding of Reily (2007) because she found out and reported that female students assessed using writing to learn methods of assessments performed better than their male counterparts. The findings of the study also suggest a clear benefit to female mathematics students from the use of writing as a tool for learning mathematics.

➤ *Qualitative Findings*

Paralleling quantitative data collection, this study also involved the collection and interpretation of qualitative data utilizing an approach referred to as template analysis, and narrative examples to illustrate problem-solving processes, utilization of flash back strategies and meta-cognitive functioning changes and growth.

• *Problem-Solving Processes that Measures the Academic Achievement*

The qualitative findings of hypotheses one and two, looking at student's writing sample for each score using a rubric designed by the researcher on the five problem-solving processes that measure the academic achievement: development of problem-solving skills, increasing conceptual understanding, demonstration of procedural application, demonstration of mathematics reasoning and development of content connections, revealed that the writing samples of students who tended to do well when providing their solutions to problems consistently scored threes and fours using the rubric. These students were successfully able to demonstrate the five problem-solving processes as they solved problems over the entire duration of the study. Also students had lower scores in conceptual understanding when compared to other processes. However, not all students were successful in this manner.

Utilizing a method involving writing provided a complete picture of student understanding when viewing the different problem-solving processes and enabled the researcher to recognize that majority of the students showed tremendous growth in their conceptual understanding as the study progressed. Requiring students to write as they solved problems provided them and the researcher a complete picture of their understanding. Student's conceptual understanding of mathematical ideas was the problem-solving process that became apparent and relevant. Thus, the researcher was aware of student's level of understanding and better assisted them with feedback that would help them be more successful. The feedback was written comments made on student's written responses to the problems that they solved. These comments led to discussions initiated by the students to correct their misunderstandings. By knowing exactly where they were making mistakes and then being able to correct the mistakes led to students enjoying the class more.

A major finding of this study is how the rich descriptions of student writing provided the researcher with a complete representation of student understanding. This resulted in the researcher being better equipped to help students become more successful in mathematics. The conceptual understanding of mathematical concepts for students is not always visible or easy for teachers to recognize when grading multiple-choice tests or short-answer tests primarily involving calculations. The use of writing as a communicative tool for students enabled the researcher to better see when students truly grasp mathematical concepts. This supported one of the major findings of this study that there was a significant difference between conceptual understanding and problem-solving ability. Through discussions with educators, many of them believe that students have a difficult time solving problems in mathematics when conceptual understanding is limited. This study shows that students may have a limited understanding conceptually while still being able to solve a problem.

The role of writing in a mathematics classroom provided the researcher with a tool that enabled the researcher to identify conceptual understanding for students as they solved problems. It is the process of writing that required students to better understand the concept that is being learned. For students to sufficiently articulate mathematical concepts well in written form, they need to have a deeper understanding of those concepts. The role of writing helped students realize what they know and did not know. This supports the idea that teachers in mathematics classrooms should implement some form of writing. The benefits of requiring students to write about their conceptual understanding regarding specific content are too significant to be ignored. Another finding revealed a significant difference between using the rubric of conceptual understanding and that of problem-solving ability. These results suggest that students received a significantly lower score in mathematics achievement for conceptual understanding compared to problem-solving ability. This finding emphasizes the importance of conceptual understanding of topics for students and suggests that the problem-solving ability of students, will only increase as their conceptual understanding improves. Problem posing is a method that can improve conceptual understanding for students in a mathematics course.

This findings agrees with the findings of Baxter et al., (2005) who stated that writing in Mathematics can help student's develop conceptual understandings and problem-solving skills. In the long term, writing in Mathematics may even help low-level students who would otherwise avoid the subject to develop long lasting Mathematical connections. Along the same lines, Santos and Semana (2015) studied expository writing in a group of eighth graders. The study analyzed interpretation, representation, and justification in student's formative assessments. The authors found that, with practice, eighth graders gradually increased their justifications and representations and decreased using vague representations. This study suggests that the combination of expository writing and feedback, can promote positive development in student's Mathematical reasoning.

- *Use of Flash Back Strategies to Improve Problem-Solving Performance*

The hypotheses one and two sought to investigate the extent to which students reported "flash back" strategies when completing the ASW's. The researcher used the three "think aloud" transcripts and the focus group transcripts to answer this question because the student's written work alone was not sufficient to determine when "flash back" was occurring. When looking at the student's written work on the first week's intervention questions, it did not appear they were using "flash back" strategies at all. It was not until the first focus group that the researcher began to understand why the students were having difficulties. In the focus group, however, the students made it clear that they did understand the context but that it was the open-ended format of the question that confused them. Obviously, correct problem identification initiates a straightforward strategy, while incorrect problem identification results in unproductive strategies. Thus students need to be taught how to approach open-ended questions. If open-ended questions were used more frequently to assess them, then students would be more apt to offer thoughtful solutions from varying perspectives.

The relationship between the student's reported use of "flash back" strategies and their performance on ASW activities, saw the most common types of "flash back" utilized were at the lowest three levels of BRT; remembering, understanding, and applying, and perhaps that is why their performance on the ASW activities at the beginning was poor. In addition, "flash back" at any of the lower three levels is considered weak "flash back", and "flash back" at any of the upper three levels, analyzing, evaluating, and creating is considered strong "flash back". Thus, strong "flash back" occurs in the context of higher-order thinking. Additionally, it's the strong "flash back" that results in successful problem solving attempts. The majority of the students were able to do strong flash back strategies, and few of them were at the lower levels of Bloom's Revised Taxonomy, so they were not helpful in getting a correct solution.

This finding supports the idea that using multiple solution methods leads to better performance, greater understanding, and improved error analysis (Hwang, Chen, Dung, and Yang, 2007; Herman, 2007; Huntley and Davis, 2008). In addition, even when students were using "flash back" effectively, and were not able to communicate it in writing on their papers, resulting in a low score, they did not put up negative attitude towards the problem solving process. Despite the problem solving difficulty each week, the student's attitudes toward participating in the intervention seemed to improve and they were more engaged in the class discussions over time, and they tried harder on the ASWs by writing more down. Findings of this study differ from Lee (2009), who found that students who flashed back more, with respect to either degree or frequency during ASW activities, tended to improve more from pretest to posttest. The findings of this study indicate that only the degree of "flash back" increased ASW performance if it was at the upper three levels of BRT. Students who "flashed back" frequently, but only at the lower three levels of BRT did not perform well on the ASW activities.

- *Meta-Cognitive Functioning Changes and Growth*

To examine the effects of Writing to Learn Mathematics from another perspective, students utilized various writing activities which were to engage students in individual reflective writing as part of the course. The intent was to explore the nature of student's individual metacognitive functioning and the ways it may have changed during the course of this study. Using the Mathography all students completed the first week of the course, the template included more specific levels that directly tied writings to conceptual connections and development, thoughts about changes students made as learners from the beginning to end of the course, and what value they saw if any in using writing to enhance self-reflection. With these levels of coding, it allowed for a more detailed and in depth analysis of all student's writing included in the course.

The process of coding with template analysis uncovered that many students did make significant changes to their approach to learning, and they were able to make deeper and meaningful conceptual connections. It also was apparent that writing in mathematics and about mathematics encouraged students to reflect on what they were learning, and allowed them to make more meaningful connections about the content and themselves as learners. Within the changes as a learner construct, majority of the students stated they experienced substantive changes as a result of the intervention.

Writing to Learn Mathematics and their comments demonstrate how some achieved a new level of sophistication as learners. Within the reflection and writing construct, students examined conceptual connections tied to quadratic equation. The student's quotes from the coded excerpts demonstrated multiple and consistent connections pulling together various facets of the course which utilized the quadratic equation. Within the value of writing construct and again based on student's comments, the researcher determined that majority of the students saw value in writing as it related to their ability to reflect on what they had learned between class sessions and over the course of the semester.

The intent of the study was to use writing as an avenue to encourage students to reflect on connections across course concepts at a deeper level and to help students learn the importance of self-concept, interest, attitudinal change, value and motivation, when it comes to their learning. Not all of the students who were involved in the study found value in the intervention. Few study participants saw little to no value in writing and did not feel that writing in mathematics encouraged them in any form of self-reflection nor did it encourage connections across course concepts. Most of their responses did not provide detail as to why and on the whole their comments were quite brief. It can be determined they viewed the intervention as a course requirement to be met rather than an opportunity to expand and enhance their approaches to learning.

Based on the frequency and insights detailed within student comments, Writing to Learn Mathematics have a profound effect on students as learners and demonstrated both changes and growth in meta-cognitive functioning as a direct result. The vast majority of students involved in the study stated in more than one construct that writing in the course had a positive effect on who they are as learners. One can also infer Writing to Learn Mathematics also improved their overall achievement in the course. The quotes from students indicate they realized learning is more than just a score on an examination. They were able to look past the score and determine if they truly understood the concepts by recognizing they can not only "do" the mathematics but "explain it" by "making meaningful connections". The results of this study are consistent with the finding of Palma (2018) and a meta-analysis conducted by Bangert-Drowns, Hurley and Wilkinson (2004) that focused on Writing to Learn. The meta-analysis included 48 school-based Writing to Learn interventions. This meta-analysis found that writing can have a positive impact on conventional measures of academic achievement. Also determined in the meta-analysis which support the results of this researcher's findings was the use of meta-cognitive prompts factors that resulted in enhanced effects of the intervention Writing to Learn. The Meta-cognitive prompts which asked students to "reflect on their current knowledge, confusions, and learning process proved particularly effective". In conclusion, this study revealed the nature of student's individual metacognitive functioning at the start of the semester and demonstrated powerful and compelling changes which occurred during the course of this study as a result of the intervention Writing to Learn Mathematics.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter deals with a brief summary of the entire work, the conclusion drawn from the findings of the study, the Educational implications of the study and the recommendations based on the findings. The limitations of the study and finally, suggestions for further research were also highlighted.

➤ *Summary of the Study*

The purpose of this study was to investigate the effectiveness of using two intervention writing to learn and Triangulation evaluation approach on Colleges of Education student's academic achievement and affective outcome in Mathematics. This included the basic general mathematics IV topics, which are variation, linear equation, simultaneous equation, quadratic equation, graph and statistics, as contain in the year two course outline. In the study academic achievement and affective outcome of year 2 students in each of these subtopics was measured through the Mathematics Achievement Test MAT survey and writing prompt. The following section includes a discussion of the results related to each of these achievement tests.

The study investigated the effectiveness of Writing to learn and Triangulation-based assessment / instruction and Traditional assessment method on academic achievement and affective outcome of students. Some students were taught the sub- topics in basic general mathematics IV for year 2 students in South-south Zone of Nigeria. The study was guided by nine research questions and nine null hypotheses were formulated and tested at 0.05 level of significance.

The study employed a non-randomized control group pretest-posttest quasi-experimental design. The population comprised 10,648 year two students found in thirteen (13) Colleges of Education, in the South-South states in the 2018/2019 academic session. Then was further focused on three thousand three hundred and six (3,306) year two students in Akwa Ibom State College of Education.

The study sampled three hundred and eighty-six (386) year 2 students from the AKS College of Education drawn using simple random sampling and purposive sampling, had control and experimental groups. Two intact classes from school of art were taught topics from Basic General Mathematics IV for sixteen weeks.

Data on Mathematics achievement and affective outcome were obtained using Mathematics Achievement Test (MAT), self-concept/ interest inventory and writing prompts. The instruments and lesson plan constructed were face and content validated by experts in the measurement and mathematics field. The reliability of instruments was established using percentage agreement rate which was found to be 88% overall exact agreement and 100% overall adjacent agreement. Mean and Standard Deviation were used to provide answers to the nine research questions posed while the Multivariate Analysis of Covariance (MANCOVA) was used for hypotheses 1,2,3,6,7 and 9, Multivariate Analysis of Variance (MANOVA) used for hypothesis 8 and Multiple Regression Analysis was used to test hypotheses 4 and 5.

The study showed that Alternative assessment -based instructional group had the highest mean gain in achievement and affective outcome. Alternative assessment -based instructional package enhanced the academic achievement and affective outcome of mathematics students more than Traditional assessment method; there was a significant effect in achievement and affective outcome amongst students taught and assessed in mathematics with Writing to learn and Triangulation approach and Traditional assessment method in favour of the first group.

➤ *Conclusion*

From the result obtained in the investigation into the effectiveness of using writing to learn as both an assessment tools and instructional method in Mathematics classrooms and the use of triangulation method of assessment on students academic achievement and affective outcome in mathematics, the following conclusions are drawn. Writing to Learn and Triangulation group had the highest mean gain in achievement (27.92/ 30.50) and affective outcome (24.40) while the Traditional group had the least mean gain score (4.51) for both achievement and (2.14) for affective outcome. A significant main effect was observed for assessment tools $F(1, 382) = 186.60$ and 1664.595 , $p < 0.00$ as well as for assessment methods $F(1, 382) = 588.93$ and 3340.54 , $p < 0.00$ for achievement and affective outcome respectively. This reveals that significant difference exist between the achievement scores of students assessed with writing to learn tools, triangulation method and those assessed with the traditional method in favour of the writing to learn tools and triangulation method group.

The writing to demonstrate knowledge and performance-based assessment tools which are Writing to Learn assessment tools as well as Reciprocity and Project -based assessment methods which were used as Triangulation assessment methods had a pronounced differential effect on mathematics achievement and affective outcome of students. The Traditional close-ended test method had no pronounced effect on student's achievement and affective outcome in mathematics. The results of the study therefore provides empirical evidence that the use of Writing to learn and Triangulation assessment approach enhanced student's achievement and affective outcome in mathematics concepts taught and assessed than the use of Traditional Closed-ended test method. Writing

to learn and Triangulation of assessment tools and methods can therefore serve as a viable alternative or supplement to conventional Closed-ended test method of teaching and assessing mathematics.

This study attempts to remedy the lack of information concerning the use of writing to learn and triangulation method of assessment as a holistic approach to assessment of the students cognitive, affective and psychomotor domains of learning achievement. Also as a means of igniting interest , improving self-concept for learning Mathematics in students as well as being used as assessment for, of and assessment as learning for the lecturers. This study is important because it provides a tool for educators who are searching for ways to improve the mathematics achievement and affective outcome of female students and students who struggle with mathematics, because the findings revealed that female student performed better than their male counterparts. The idea of triangulation has been in the assessment literature for at least two decades.

There are two different ways of thinking about it that have informed this study. First, Thomas, Lightcap and Rosencranz (2005) address its value in assessing general education: Triangulating methods of analysis is commonly recommended to overcome validity problems. The idea is a simple one; when multiple threats to validity of measures emerge, use multiple sources of data generated by multiple methods of analysis to meet them. If the different methods seem to lead to similar conclusions, then the level of uncertainty is reduced. From this perspective, then, triangulation helps with the formal requirements for good assessment and evaluation. By using multiple methods, one reduces reliance on any single measure that may be, and usually is, inherently flawed.

➤ *Educational Implications of the Study*

The results of this study demonstrate a number of ways in which writing can be a successful mathematics-teaching/ assessment tool. The study also reveal that varying the method of assessing students and thereafter triangulate the result , helps lecturers to assessed the students in the three domains of learning as well as validate the scores allotted to the students. The present study provides mathematics educators with new ideas on how to use writing/ triangulation as a teaching/assessment tool and methods.

The results also give student's positive perceptions on the value of writing in mathematics classes. In addition, the results give a strong indication that writing is a useful way to close the achievement gap between male and female students in mathematics. This study also shows that writing is a task which can help students who struggle in mathematics. Thus this study provides strong support for the argument that mathematics educators should make writing a regular part of their pedagogy.

The curriculum planners should bear in mind the integration of writing to learn and triangulation of assessment methods when organizing mathematics curriculum. In this current era of high stakes testing, it is extremely important that disciplines within the academy support one another. Mathematics departments need to depend on English departments in order to reach the standards that have been set by forces beyond their control. English departments are in the same situation. All students need to be able to write open-ended prompts in both arts and sciences (mathematics). In addition to this, students also need to have met multiple anchors that are assessed at each level of education. This study shows that if educators wish to effectively prepare students for these standardized assessments, it is important that teachers use writing and mathematics simultaneously.

Increasing the use of writing, policy makers in education will be expected to utilize the information provided by this research as a basis for considering the best assessment/ instructional techniques to be used in mathematics teaching programmes. Therefore, their decision should play down on the use of Traditional (Close-ended) assessment method recommended for mathematics assessment and teaching.

The overall implication for the government is the need to provide schools with adequate facilities, such as computers, projectors etc. teaching aids and models that will guide the teachers towards using writing to learn in their teaching and subsequently the assessment of students. Also, the materials that will be provided by the government will help both the teachers and students in preparing their portfolio and performance-based activities useful for mathematics teaching/ learning and proper assessment.

➤ *Recommendations*

Based on the result of this study, the following recommendations were made;

- Mathematics lecturers should be encouraged to employ the use of assessment/ instructional activities such as writing to learn,, performance-based alternative solution worksheet, writing to demonstrate knowledge and Triangulation Reciprocity and Project-based methods of assessment before, within and after a Mathematics lesson in order to relate Mathematics to real life. This will help to correct the impressions the students have about Mathematics, and help in retention of Mathematics concept taught.
- To ensure that Mathematics educators are equipped with the usage of writing to learn and triangulation assessment methods, pre-service Mathematics educators should be trained on the use of alternative method of assessment during their training process by the teacher educators.
- Tertiary institutions, through the Ministry of Education policy makers, should organize seminars, workshops and conferences on the use of the Alternative assessment methods/ instructional package for serving educators, teacher educators, textbook writers and curriculum developers.

- Curriculum planners should incorporate the use of writing to learn and triangulation assessment methods/ instructional package in restructuring Mathematics curriculum in Nigeria. The curriculum should be restructured to reflect the basic concept of the Alternative assessment/ instructional package as they pertained to Mathematics teaching/learning and assessment.

➤ *Limitations of the Study*

The potential generalizability of the findings may be limited by a number of factors which could not be avoided.

- Not only would a larger sample and longer duration have provided better data, but having University and Polytechnic students would have enhanced a better generalizability of the findings.
- Furthermore, had the lecturer, other than the regular course lecturer, taught both the experimental and control groups, it would have added to the credibility of the findings and ruled out any potential shades of bias.
- However, the experiment was conducted in one state, Akwa Ibom States in the South- South Geopolitical Zone of Nigeria, because some areas were inaccessibly remote for anyone from another area. This accounted for not finding any volunteers to teach the groups, hence the regular course lecturer ended up teaching both groups. In spite of these limitations, the findings remains valid.

➤ *Suggestions for Further Research*

This study in itself is not exhaustive in terms of scope as there are other areas that could not be researched into under the usage of alternative assessment methods. It is rather an attempt to stimulate and facilitate further research in this direction of academic endeavour. Based on the findings and limitations of this study, the researcher suggested the following research:

- A study is suggested on the effectiveness of writing to learn and Triangulation assessment methods/ instructional package on the retention of mathematics concepts.
- A study is suggested that will use students from university, polytechnic and colleges of Education to replicate this study. In this case, a larger sample and longer duration will provide better data and enhanced a better generalizability of the findings.
- This study focused on Variation, Linear equation, Quadratic equation, Simultaneous equation and Statistics topics on year II Basic General Mathematics, thus the results cannot be generalized to the other mathematics courses and other mathematics topics. Further research should be conducted at different class level and different aspect of mathematics.
- This research was limited to one semester. Further research should be conducted to investigate long term effects of Alternative assessment usage on student's achievement and affective outcome in all algebra, Geometry and calculus content areas.
- The topics of equation are mostly related with other subjects like Sciences. Therefore, the effects of the Alternative assessment methods usage on Science courses should be investigated and comparison with the usage on basic general mathematics should be a further research issues.
- This study was limited to two tools, writing to demonstrate knowledge, performance –based and reciprocity with peer and project-based methods, in order to understand the effects of dynamic mathematics assessment method in depth, using other alternative approaches such as Flipped classroom model, Portfolio-based method, open-ended test method, Crib sheet, Frayer model and others are suggested.

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APPENDICES**APPENDIX A I**

Population Distribution of the Year Two Students of the AKS Colleges of Education in the 2018/2019 Academic Session

SCHOOL	NO OF MALE	NO OF FEMALE	TOTAL
A k w a I b o m S t a t e C O E	1 , 2 8 6	2 , 0 2 0	3 , 3 0 6
T O T A L	1 , 2 8 6	2 , 0 2 0	3 , 3 0 6

APPENDIX A II

Sample Size Distribution of the Groups

Name of Departments in experimental		No Of Student	No Of Students in control		Total
Male	Female		Male	Female	
S o c i a l s t u d i e s	4 3 5 4		4 6 5 4		1 9 7
P o l i t i c a l s c i e n c e			4 8 4 4	5 1 4 6 1 8 9	
TOTAL	9 1	9 8	9 7	1 0 0	3 8 6

APPENDIX A III

Table of Specification for Mathematics Achievement Test

C o n t e n t	Remember 0 4 %	Understand 0 8 %	Application 2 0 %	Analysis 1 6 %	Evaluation 2 0 %	Creating 3 2 %	T o t a l
V a r i a t i o n 2 8 %	1	1	1	1	1	2	7
L i n e a r E q u a t i o n 1 6 %	0	0	1	1	1	1	4
Quadratic Equation 2 0 %	0	0	1	1	1	2	5
Simultaneous Equation 1 2 %	0	0	1	0	1	1	3
S t a t i s t i c s 2 4 %	0	1	1	1	1	2	6
T o t a l	1	2	5	4	5	8	2 5

APPENDIX B I: MATHEMATICS SELF-CONCEPT QUESTIONNAIRE

This inventory is designed to help the researcher gain some insights on how effective various aspects of your Mathematics classes are for you as a student. You are being asked to identify the methods you feel are the best for learning Mathematics. Your responses will help Mathematics Lecturers improve the Mathematics curriculum in order to enhance student's learning experiences. Please answer all items as completely and honestly as you can.

➤ *Demographic Information: Please Write Legibly*

Name: _____

Email: _____

- Gender (check one) ____ Male ____ Female
- Name of institution _____
- Course of study: -----
- Year of study: _____
- Department: _____
- Semester: _____

➤ *Instruction:*

Against each statement below, there are four options, tick (☐) the one that best expresses your feelings to the statement made on Mathematics learning. There is no right or wrong answer.

➤ *Keys:*

- Strongly Agree (SA), Agree (A), Disagree (D), Strongly Disagree (SD).

Mathematics Self-Concept Questionnaire

S / N	I	T	E	M	S	S	A	A	D	S	D
1	Mathematics is a challenging subject and I dislike it.										
2	I work to the best of my ability in Mathematics class.										
3	Mathematics assignments helps me understand Mathematics concepts better.										
4	I find most concepts thought in Mathematics class difficult.										
5	The Mathematics scores I earn reflects my efforts										
6	I believe I can understand the content in a mathematics course better, if my Lecturer explains a practical application of the concept we are studying.										
7	I am sure I can understand Mathematics.										
8	I feel confident enough to ask questions in my mathematics class.										
9	I get nervous when taking a mathematics test.										
1 0	I worry that I will not be able to complete every assignment in a mathematics course.										
1 1	I get tense when I prepare for a mathematics test.										
1 2	I feel stressed when listening to mathematics instructors in class.										
1 3	I worry that I will not be able to use mathematics in my future career when needed.										
1 4	I feel shy to ask the lecturers for help when I can not solve a problem in mathematics.										
1 5	I feel confident solving Mathematics problem when we are in a group.										
1 6	I am usually nervous about the next Mathematics course I have to take.										
1 7	I believe that doing even more writing activities in Mathematics classes would improve my learning/understanding.										
1 8	I believe I can complete all of the assignments in a mathematics course if my Mathematics Lecturer is available and willing to help me.										
1 9	I believe I can understand the concept taught better when my Lecturer solves more examples.										
2 0	I am curious about discoveries in Mathematics.										
2 1	I enjoy discussing Mathematics with my friends.										
2 2	When I am able to solve Mathematical problems, it increases my self-esteem.										
2 3	I enjoy solving more problems when the requirement of each question is clear and straight forward.										
2 4	My creativity in solving Mathematical problems increases with the Lecturer's use of several methods to solve Mathematical problems.										

2	5	The use of different instructional/ assessment strategies Offers me greater flexibility in solving Mathematical problems.				
2	6	I believe I can learn more mathematical concepts in a class that uses writing activity than a class that does not use writing activity.				
2	7	I believe I can understand Mathematics concepts better if I have to write about them at the end of a unit.				
2	8	Knowing Mathematics will give me a career advantage.				
2	9	Mathematics is important for everyday life and can help me find solutions to everyday problems.				
3	0	Mathematics helps to develop the mind and makes me an independent thinker.				
3	1	I feel that I will be able to do well in future mathematics courses.				
3	2	I believe that one way I can improve my learning of mathematics is by doing more writing about mathematics concepts.				
3	3	I believe Mathematics will play a very big role in my future?				
3	4	I feel confident when taking a mathematics test that I spent much time to prepare for.				
3	5	I believe that learning mathematics has much relevance in my life.				
3	6	I would prefer my lecturer to assesses me using varieties of assessment methods.				
3	7	I try to solve difficult problems of mathematics myself rather than ask others, when my lecturer writes comments on my assignments.				
3	8	Some of my Mathematics assessments involve writing activities				
3	9	I completed Basic General Mathematics II GST course in my year one in the college with very good grade and this boost my self-esteem.				
4	0	Completing writing assignments like Journal Entries on Mathematical concepts, Learning Logs for Mathematical concepts/connections, Threaded Discussions related to study skills or Mathematics concepts, Essays about Mathematics Concepts, Narratives of Mathematical Steps in Mathematics courses makes me more confident.				

APPENDIX B II: MATHEMATICS INTEREST INVENTORY

S / N	I	T	E	M	S	S	A	A	D	S	D
1	Mathematics is my favourite subject, and I enjoy Mathematics class.										
2	Mathematics should be a compulsory subject.										
3	Mathematics period makes me nervous.										
4	I am always under strain in a mathematics class.										
5	I try to understand very attentively the concepts taught by the lecturers in the class.										
6	I find it very difficult to solve problems in mathematics.										
7	I am very afraid of mathematics lecturers.										
8	I always go well prepared to mathematics class.										
9	Am happier in a Mathematics class than in any other class.										
1 0	The way my Mathematics Lecturer's teaches makes me to enjoy mathematics better.										
1 1	I think I can handle more difficult Mathematics problems if my Lecturer uses several methods.										
1 2	My Mathematics Lecturer uses a variety of activities to help me learn.										
1 3	I leave my Mathematics class feeling that I understand the lesson.										
1 4	Writing in mathematics class has greatly changed the way I learn Mathematics.										
1 5	I feel insecure in mathematics class.										
1 6	I have good feeling for mathematics.										
1 7	I face difficulty in doing lengthy problem solving procedures of mathematics.										
1 8	Mathematics is not my favourite subject.										
1 9	Mathematics is of no use in one's life.										
2 0	I practice mathematics problems in my leisure time.										
2 1	I never want to be absent in mathematics class.										
2 2	Mathematics is an obstruction in student's all-round development.										
2 3	I would prefer to read story books than doing mathematics.										
2 4	I try to complete mathematics problems at home that were half done at school.										
2 5	I would prefer to memories spellings in English than solving mathematics problems.										
2 6	I read the columns containing data in news paper with great interest.										
2 7	I study mathematics that are more related to other subjects.										
2 8	I try to solve difficult mathematics problems by myself rather than ask others for help.										
2 9	I would prefer to take up a career in mathematics in future.										
3 0	I would never suggest to my friends to study courses.										
3 1	I would like to solve oral mathematics in my leisure time.										
3 2	I solve quizzes related to mathematics published in papers and magazines with great interest.										
3 3	I become irritated if anyone wants to learn mathematics from me.										
3 4	I am afraid of failing in mathematics examination.										
3 5	Most of my friends are good in mathematics.										
3 6	I find mathematics period boring.										
3 7	I feel happy, getting more marks in mathematics than other subjects.										
3 8	I feel happy when my mathematics lecture is around for class.										
3 9	I feel more hard work can make me a good student of mathematics.										
4 0	I have difficulty in concentrating while doing mathematics.										

APPENDIX C: PRE- LEARNING BIOGRAPHY(MATHOGRAPHY)

The first writing assignment for the term is autobiographical (Mathography) essay titled, My Mathematics Learning Biography: Who Am I?

➤ *The Following Should be Addressed*

- Who you are as a Mathematics student
- Feelings you have toward Mathematics in general
- Your strengths and weaknesses as a Mathematics student
- Your goals for this course and how you monitor your growth in mathematics learning.
- Your long term educational goals and how you intend to achieve them.
- Why you feel you will be successful in this course Do not write the grade you want to earn. Write about who you are as a learner.
- Write about the methods your lecturer uses to assess you and how it has helped you to learn mathematics.

APPENDIX D: END OF SEMESTER ESSAY (POST-MATHOGRAPHY)

The overall theme of this essay is the growth you have experienced as a learner over this semester in this course. Refer to in class activities, course projects, the opportunity to work with other students, the “Think aloud” discussions, your performance and your progress in the course to demonstrate that growth. Please address the following questions.

- What is your vision of a successful learner? Do you feel that you have accomplished your goals this semester? Explain what your goals were and why you feel you met or didn’t meet your goals. I do not want to hear about the grade you expect but what you learned about yourself as a learner.
- How have you changed as a learner this semester? You were asked to “do some different things in the basic general Mathematics class” in this course that were intended to help you to grow as a learner. Did you change at all in terms of how you approach learning Mathematics? Why or why not?
- Finally, do you feel that the assesement method used by your lecturer (writing to learn Mathematics) required you to reflect on what you really know, understand, and can do with respect to variation, linear, quadratic, simultaneous equations graphs and statistics? Even if you did not “enjoy” the process, do you see a connection to writing and reflecting on what you have learned? How do you expect to use this method (writing) in your future profession? Please write a detailed example for this experience. Conclude your essay with a description of how you monitor your learning “then and now”. The title of your essay should be “That Was Then, This Is Now” It is a minimum of 2 pages double space typed with 12 point font and 1 inch margins on all sides. You are required to use Times New Roman or Verdana as the font.

APPENDIX E I

➤ *Procedure for Writing to Learn Method of Instruction*

Step 1: Introduce the writing to learn and the reciprocity the methods for at least two lesson periods.

Step 2 Divide the class into groups taking cognizance of the student's mathematical ability. Year one examination grade and the pretest scores could be used for that, and the group should have low, average and high ability forming the groups.

Step 3: Review background of related experience and state the objectives of the new lesson. Explain to the student the tools and how to use them to solve problems.

Step 4: Present the learning materials in step by steps, anticipating student's errors and areas of difficulty:

Step 5: Have students work in small groups and clarify purpose - (Virtual Manipulatives helps students understand why they are doing the work and why it is important).

Step 6: Keep the students on task by providing structures. (The manipulative features provide pathways for the learners).

Step 7: Incorporate assessment and feedback: offer lecturer led feedback and provide models of expert work with the aid of writing to learn generated feedback excerpts.

Step 8: Reduce students support but gradually increase complexity and difficulty of learning materials. (Student – centered and guided discovery learning or interactive learning).

Step 9: Practice consolidation - putting all the steps together and check for students mastery. (Flash back).

Step 10: Provide independent practice, through investigation of conjectures with project based problems.

Step 11: Facilitate application to new situation, (Connection to real world problem situations).

Step 12: Demonstrate how to cope with difficulties (if needed)

Step 13: Review major guidelines and principles for the accomplishment of the tasks.

Step 14: Invite questions and provide answers where necessary.

Step 15: Evaluate Student's Performance.

APPENDIX E II PROCEDURE FOR TRADITIONAL METHOD OF INSTRUCTION

Step 1: Introduce the lesson topic

Step 2: Review briefly the previous lesson

Step 3: State the objectives of the new lesson.

Step 4: Teach the general concepts and principles of the topics.

Step 5: Discuss and describe skills used in the lessons.

Step 6: Assess the students on the spot.

APPENDIX E III: LESSON PLAN FOR TRADITIONAL METHOD GROUP**Subject:** Basic General Mathematics IV**Class:** year 2**Topic :** Variation**Duration:** 1 Hour**Course Code:** GSE 222➤ *Instructional Objectives*

At the end of the lesson the students should be able to:-

- Solve correctly with ease and 75% success some problem involving direct variation.
- Solve with ease and 75% success some problem on inverse variation.
- Solve with ease and 75% success some problem on joint variation.
- Solve correctly with ease and 75% success some problem on partial variation.

➤ *Entry Behaviour*

- *The Students have Known:-*

- ✓ Solving equation involving fractions.
- ✓ Change of subject of formula.
- ✓ Solving simultaneous equation in two variables.

➤ *Instructional Material*

A chart showing solved problems on variations and reciprocal table.

➤ *Content Development*• *Introduction:*

The word variation is used to describe the relationship which exist among two quantities. In everyday life we come across relationships between quantities. These relationship can be seen in terms of varying one quantity to another. For instance if a worker is paid a certain rate for every hour that he works, the total wages due to him in a given period will depend on the hours he works. Hence a kind of variation exist.

Content Development	A c t i v i t i e s		Strategies
	L e c t u r e r s	S t u d e n t s	
Step 1: Direct variation <small>When the value of one variable is directly proportional to the value of another variable, the two variables are said to be in direct variation. This can be written as $y \propto x$ or $y = kx$ where k is the constant of variation.</small>	S o l u t i o n : Form the variation equation using the given variables, $C = KL + 50$. Solve for C algebraically or graphically, elimination or substitution to have #65.00.	They solve along with the lecturer and solve others in their notebook following the teacher's example.	Set induction and use of examples
Step II: Inverse variation <small>When the value of one variable is inversely proportional to the value of another variable, the two variables are said to be in inverse variation. This can be written as $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$ where k is the constant of variation.</small>	1. The constant of variation, k , is $6 \times 12 = 72$. 2. Since 4 are absent, that leaves 8 workers ($12 - 4 = 8$). 3. $Y = 72/x$, $y = 72/8$, $x = 8$ therefore $y = 72/8 = 9$ hrs.	Listen and participate in solving along with the teacher. They expand the given expressions in their notebook.	Explanation and use of examples
Step III: Joint variation <small>When the value of one variable is proportional to the value of two or more variables, the two variables are said to be in joint variation. This can be written as $y \propto xz$ or $y = kxz$ where k is the constant of variation.</small>	E x e r c i s e I I I : S o l u t i o n :	Solve the exercise and do the classwork given by the lecturer in their notebook.	Use of examples, planned

[illegible]

➤ *Summary*

In our lesson we have learnt that the word variation is used to describe the relationship which exist among two quantities. In everyday life we come across relationships between quantities. That when the ratio of a variable x to another variable y is always constant, then x is said to vary directly as y . If two variables x and y are such that when x increases y decreases or vice versa, the two variables are said to be inversely proportional. That when the value of a quantity is proportional to two or more other quantities, it is said to be joint variation And lastly that partial variation has to do with quantity or variable that depends on another independent variable, where one of the independent variable remains the same always and the other changes according by the quantity of independent variable used.

➤ *Assignment*

- There are 14 boys and 16 girls in Mrs Uko's class. What ratio best represent the relationship between the number of boys and the total number of students in Mrs Uko's class. (a) 7/15 (b) 7:10 (c) 7/9 (d) 14:16 (e) 8:15. (1b). If the ratio of A to B is 3:4, and the ratio of B to C is 2:3, what is the value of A when C is 5. (a) 10 (b) 2/2 (c) 5/2 (d) 5 (e) 6.
- The volume of wood in the tree (V) varies directly as the height (h) and inversely as the square of the girth (g). If the volume of the tree is 144m^3 when the height is 20m and the girth is 1.5m. What is the height of a tree with a volume of 1000 and girth of 2m.
- If 14 women can cultivate 42 acres of land in 18 weeks, how many weeks will it take 21 women working at the same rate to cultivate 52 acres of land.

➤ *Lesson Plan for Linear Equation*

Subject: **Basic General Mathematics IV**

Class: Year 2

Topic : Linear Equation

Duration: 1 Hour

Course Code: GSE 222

➤ *Instructional Objectives*

At the end of the lesson the students should be able to:

- Find with ease and 75% success L.C.M of algebraic fraction.
- Add and subtract fractions with monomial denominators with ease and 80% success.
- Solve correctly with ease and 80% success some problem involving linear equation.

➤ *Entry Behaviour*

The students have known: (1) Addition and subtraction of numbers (2) multiplication and division of numbers (3) solving algebraic expression.

➤ *Instructional Material*

A beam balance is use to illustrate the idea of equation.

➤ **Content Development**

An equation is a mathematical statement that shows equality between two expressions. Equation which arises out of practical problems in many ways, usually consist of more or less complicated expressions on both sides of the equation. There are different forms of equations, one of them being linear equation. A linear equation is a mathematical statement that contains only term in one variable and constant, and the highest power of the variable is 1.

Content Development	A c t i v i t i e s		Skills Emphasized
	L e c t u r e r s	S t u d e n t s	
Step I: Lowest common multiples of algebraic expressions <small>The lowest common multiple of two or more algebraic expressions is the smallest algebraic expression which is exactly divisible by each of the given expressions. For example, the L.C.M. of $4x$, $8x^2$, $16x^3$, $20x^4$ is $40x^4$. It can be seen that amongst the common multiple that 12 is the least.</small>	Ex. I: Find the L.C.M. of the number (1) 9, 6 and 3 (2) x^2, y and Zy. S o l u t i o n (1) $9 = 3 \times 3$ $6 = 3 \times 2$ $3 = 3 \times 1$ L.C.M = $3 \times 3 \times 2 = 18$ $x^2 = x \times x$ $y = 1 \times y$ $Zy = Z \times y$ L.C.M = $x \times x \times y \times Z$ $= x^2 y Z$	<small>They solve along with the lecturer and solve (1) and (2) in their note book following the lecturer's example.</small>	Set Induction and Use of examples
Step II: Simple Equations <small>Equation involving one variable and one or more terms of the variable is called a linear equation. For example, $2x + 3 = 7$ is a linear equation. The solution of a linear equation is the value of the variable which satisfies the equation. For example, the solution of the equation $2x + 3 = 7$ is $x = 2$.</small>	<small>2. Solve $4(3x - 2) = 6$</small> 2. $4(3x - 2) = 6$ $4 \times 3x - 4 \times 2 = 6$ $12x - 8 = 6$ Add 8 to both sides $12x - 8 + 8 = 6 + 8$ $12x = 14$ Divides both sides by 12 $\frac{12x}{12} = \frac{14}{12}$ $x = \frac{14}{12}$	<small>Listen and participate in solving along with the lecturer. They expand the given expressions in their note book.</small>	Use of examples
Step III: Simple Equation Involving Algebraic Fraction <small>A simple equation involving algebraic fraction can be solved as ordinary simple equation except that L.C.M. should be used to multiply through to clear the fraction, if so which.</small>	Ex. III: Solve the following (1) $\frac{3}{4x} + \frac{1}{2}$. S o l u t i o n 1. $\frac{3}{4x} + \frac{1}{2} = 1$ Subtract $\frac{1}{2}$ from both sides $\frac{3}{4x} + \frac{1}{2} - \frac{1}{2} = 1 - \frac{1}{2}$ $\frac{3}{4x} = \frac{1}{2}$ Multiply through by 4 $\frac{3}{4x} \times 4 = \frac{1}{2} \times 4$ $\frac{3}{x} = 2$ Divide through by 3 $\frac{3x}{3} = \frac{2}{3}$ $x = \frac{2}{3}$	<small>Factorize the algebraic expressions on the board. They also factorize individually in their note book.</small>	Stimulus variation
E v a l u a t i o n	<small>Evaluates the students by giving them some problems on algebraic expression as exercise and assignment to solve in their note book.</small>		C l o s u r e

➤ **Summary:**

In our lesson we have learnt that an equation is a mathematical statement that shows equality between two expressions. That a linear equation is a mathematical statement that contains only term in one variable and constant, and the highest power of the variable is 1. Also that a simple equation involving algebraic fraction can be solved as ordinary simple equation except that L.C.M should be used to multiply through to clear the fraction, if so which.

➤ **Assignment: (1)**

Aman is four times as old as his son. In four years time he will be three times as old. What are their ages now?

The same number is added to both the numerator and the denominator of the fraction $\frac{7}{17}$. If the fraction then becomes $\frac{6}{11}$, find the number added.

➤ Lesson Plan For Quadratic Equation

Subject: Basic General Mathematics IV

Class: year 2

Topic : Quadratic Equation

Duration: 1 Hour

Course Code: GSE 222

➤ Instructional Objectives

- At the end of the lesson, the students should be able to: Solve correctly quadratic equation by factorization method with ease and 85% success.
- Solve with ease and 70% success quadratic equation of the type $(ax + b) = c$ where a, b, c are real numbers and x a variable.
- Solve quadratic equation with imaginary roots with ease and 75% success.
- Solve correctly with ease and 75% success some problem on quadratic equation by using quadratic formula.

➤ Entry Behaviour

The students have known: (1) Factorization of quadratic expression (2) simplification of quadratic expression.

➤ Instructional Materials

A flip chart showing the steps to be taken when solving quadratic by completing squares.

➤ Content Development

• Introduction:

A quadratic equation is an equation of second degree, that is an equation in which 2 is the highest power of the unknown.

Content Development	Activities		Skills Emphasized
	Teachers	Students	
Step I: Solving Quadratic Equation By Factorization Factorize quadratic expression using splitting middle term method to get quadratic expression in the form $(x + a)(x + b) = 0$. Ex I: (1) Suppose the area of a rectangular billboard is $5^2 - 11x + 6$ and the length of one of its sides is $5 - 2$. Find the value of. Factorize: (2) $4x^2 + 13x + 9$ (3) $x^2 + 11x + 18 = 0$. S o l u t i o n s The factors are -3 and 5 $\therefore (-3)(5) = -15 = -11$. Following from the latter two equations, -3 and 5 . $4x^2 + 13x + 9$, the product of 1 st and last term is $36x^2$. Factors that will give the product $36x^2$ and sum $13x$ is 9 and $4x$. $\therefore 4x^2 + 13x + 9 = 4x^2 + 4x + 9x + 9$ $4x(x + 1) + 9(x + 1)$ $(x + 1)(4x + 9)$	Ex I: (1) Suppose the area of a rectangular billboard is $5^2 - 11x + 6$ and the length of one of its sides is $5 - 2$. Find the value of. Factorize: (2) $4x^2 + 13x + 9$ (3) $x^2 + 11x + 18 = 0$. S o l u t i o n s The factors are -3 and 5 $\therefore (-3)(5) = -15 = -11$. Following from the latter two equations, -3 and 5 . $4x^2 + 13x + 9$, the product of 1 st and last term is $36x^2$. Factors that will give the product $36x^2$ and sum $13x$ is 9 and $4x$. $\therefore 4x^2 + 13x + 9 = 4x^2 + 4x + 9x + 9$ $4x(x + 1) + 9(x + 1)$ $(x + 1)(4x + 9)$	They solve along with the lecturer and solve 1 and 2 in their note book following the lecturer's example.	Set Induction and Use of examples
Step II: Solving Quadratic Equation of the Type $(ax + b)^2 = c$ and construction with given roots. If $x^2 = 16$, then $x = +\sqrt{16} = +4$. This is true since $(+4)^2 = 16$ and $(-4)^2 = 16$. We will use the above idea to solve equations.	Ex. III: 1. Solve the equation: $4(2-3)^2 - 36 = 0$. $2(x-1)^2 = 4(2)(2x-1)^2 = 3(x+1)^2 = 9$. 4. Construct a quadratic equation given that The roots are -5 and $-2/9$. S o l u t i o n The following is an example of an algebraic approach $4(2-3)^2 - 36 = 0$ $4(2-3)^2 = 36$ $(2-3)^2 = 9$ $2 - 3 = \pm 3$ $= 3 \pm 3 / 2$ $= 3 \text{ or } 0$ $(x - 1)^2 = 4$ Take square root of both sides $X - 1 = \pm \sqrt{4}$ $X - 1 = \pm 2$	Listen and participate in solving along with the teacher. They expand the given expressions in their note book.	Use of examples

	$\therefore x - 1 = +2 \text{ or } x - 1 = -2$ $\text{i.e. } x = 2 + 1 \text{ or } x = -2 + 1$ $\therefore x = 3 \text{ or } -1$ $(2x - 1)^2 =$ <p>Take square root of both sides</p> $2x - 1 = \sqrt{3} \text{ add 1 to both side}$ $2x = 1 + \sqrt{3} \text{ divide through by 2}$ $x = \frac{1}{2} + \frac{\sqrt{3}}{2}$ <p>i.e. either $x = \frac{1}{2} + \frac{\sqrt{3}}{2}$ or $x = \frac{1}{2} - \frac{\sqrt{3}}{2}$, $\sqrt{3} = 1.73$</p> <p>So $x = \frac{1}{2} + 1.73$ or $\frac{1}{2} - \frac{1.73}{2} = 0.5 + \frac{1.73}{2}$ or $0.5 - \frac{1.73}{2}$</p> $\therefore x = 1.37 \text{ or } 0.37$ $\frac{1}{2} + \frac{1.73}{2} \text{ or } \frac{1}{2} - \frac{1.73}{2} = \frac{0.73}{2}$ $\underline{1 + 1.73} = \underline{2.73} = 1.37 \text{ or } 0.6$		
Step III: Graph of Quadratic Equation <small>Another way of solving quadratic equation is by graphical method. If it is related to x, such that there are no value of y, there is a value to be found in x. The set of values of x is called domain which contains a set of ordered numbers and a set of corresponding values of y called range of the function. To solve quadratic equation by the following steps are considered:</small> Make a table of values for the function which contains the value of x and corresponding to y Choose a suitable scale for both axes if not given Plot the point on the graph Obtain the roots of the equation which is the solution	Ex. I: Solve the quadratic equations: $(1) \quad x^2 + 8x = 0$ $(2) \quad 3t^2 - 75t = 0$ Solution $x^2 + 8x = 0 \text{ factorize LHS}$ $x(x + 8) = 0$ $x = 0 \text{ or } x + 8 = 0$ $x = -8$ $\therefore x = 0 \text{ or } -8$	Factorize the algebraic expressions in the box and record.	Stimulus Variation
E v a l u a t i o n	Factorize $v^2 - 6v - 2$ Factorize $36 - 12t +$ Solve using formula $2x^2 - 4x - 3 =$	Solve the exercise and assignment given by the teacher in their note book	C l o s u r e

➤ *Summary*

In our lesson, we have studied more about solving quadratic equation, by the formula and factor method. We also looked into Imaginary root of quadratic equations and the solution to quadratic equation such as $(ax + b)^2 = c$.

➤ *Assignment*

- Factorize $(-2)2 + (-2) + 3(-2) + 3$ completely.
- Solve the equation (a) $(2x + 1)^2 = 5$
- Solve using the formula $2x^2 + 15x + 7 = 0$

➤ *Lesson Plan for Simultaneous Linear Equations in Two Variables*

Subject: Basic General Mathematics IV

Class Year 2

Topic Solution of Simultaneous Linear equations in two variables

Duration: 1hour

Course Code: GSE 222

➤ *Instructional Objectives*

At the end of the lesson, the students should be able to solve simultaneous linear equation by (1) elimination method (2) substitution method (3) graphical method with ease and 80% success.

➤ *Entry Behaviour*

The students have known (1) linear equation in one variable and two variables (2) quadratic equation in one variable.

➤ *Instructional Material*

Chart of solved simultaneous equation by elimination, substitution and graphical methods.

➤ **Content Development**• **Introduction:**

A linear equation is the equality of two linear expressions and a constant. It is solved simultaneously to obtain the two variables. There are methods of solving these equations which are elimination, substitution and graphical methods.

Content Development	Activities		Skills Emphasized
	Teachers	Students	
Step I: Elimination Method <i>To solve simultaneous equations graphically, express one of the variables in terms of the other in both equations. Make a table of values and plot the graphs on a coordinate plane. The intersection of the two lines gives the solution.</i>	Ex. I: Using elimination method solve the following equation. 1) Solve the system of equations $2x + 3y = 12$ and $3x + 2y = 10$. 2) John solved the equation, $2x + 3y = 12$, and he got the correct answer: $x = 3$, $y = 2$. Find the value of y . Solution: 1) Let one yam cost x and goat y . 2.5 tuber of yam and 3 goats will be $(5x + 3y) = ₦220$ 15 tuber of yam and 2 goats will be $(15x + 2y) = ₦170$ $5x + 3y = ₦220$ i $15x + 2y = ₦170$ ii multiply (i) by 3 , (ii) by 1 $15x + 9y = ₦660$ iii $15x + 2y = ₦170$ iv Subtract iv from iii $7y = 490$ $y = 70$, put in (i) $5x + 3(70) = ₦220$ $5x + 210 = 220$ $5x = 10$ $x = 2$, yam = ₦2 and goat = ₦70. <i>What is the value of x in the equation $2x + 3y = 12$ and $3x + 2y = 10$? If the value of x is 3, what is the value of y in the equation $2x + 3y = 12$ and $3x + 2y = 10$?</i>	They solve along with the lecturer and solve 1 and 2 in their note book following the lecturer's example.	Set induction and use of examples
Step II: Substitution Method <i>To solve simultaneous equations graphically, express one of the variables in terms of the other in both equations. Make a table of values and plot the graphs on a coordinate plane. The intersection of the two lines gives the solution.</i>	Ex. II: What is the length of the side of a rectangle when the perimeter is 32cm and area 55cm^2 ? 2. Solve by substitution method the following (i) $x - 2y = 4$, $2x + 5y = 17$ (ii) $4x - 3y = 11$, $2x + 5y = 25$. <i>What is the length of the side of a rectangle when the perimeter is 32cm and area 55cm^2? If the perimeter is 32cm and area is 55cm^2, what is the length of the side of the rectangle?</i>	Listen and participate in solving along with the lecturer. They expand the given expressions in their note book.	Explanation and use of examples
Step III: Graphical Method <i>To solve simultaneous equations graphically, express one of the variables in terms of the other in both equations. Make a table of values and plot the graphs on a coordinate plane. The intersection of the two lines gives the solution.</i>	Ex. III: Solve graphically the simultaneous equations $x - 2y = 4$ and $2x + 5y = 17$. <i>What is the length of the side of a rectangle when the perimeter is 32cm and area 55cm^2? If the perimeter is 32cm and area is 55cm^2, what is the length of the side of the rectangle?</i>	Listen and participate in solving along with the lecturer. They solved the Given problems in their note book.	Explanation and stimulus variation
Step IV: Graphical Solution of Simultaneous One Linear, One Quadratic and Analytical Solution <i>To solve simultaneous equations graphically, express one of the variables in terms of the other in both equations. Make a table of values and plot the graphs on a coordinate plane. The intersection of the two lines gives the solution.</i>	Ex. I: What are the values of x and y in the equation (1) $3x + 2y = 12$, $xy + 5y = 21$ (2) $3x + y^2 = 22$, $2x - y = 10$ (3) $3x + 4y = 11$, $xy = 2$ <i>What is the value of x and y in the equation $3x + 2y = 12$ and $xy + 5y = 21$? If the value of x is 3, what is the value of y in the equation $3x + 2y = 12$ and $xy + 5y = 21$? If the value of x is 3, what is the value of y in the equation $3x + 2y = 12$ and $xy + 5y = 21$?</i>	Listen and participate in solving along with the lecturer in their note book.	
Evaluation	Solve the simultaneous equation by elimination method $6x + 8y = 85$, $4x - 2y = 10$ Solve the simultaneous equation by substitution $x - 2y = 4$, $2x + 5y = 17$ Solve graphically $2x - y = 3$, $x + 3y = 5$. Take values of x from -1 to 3 , use the scale of 1cm to represent 1 unit on x -axis.	Solve the exercise and assignment given by the teacher in their note book	Use of examples, planned repetition and Closure

➤ **Summary**

In our lesson, we have learnt that simultaneous equations in two variables can be solved by elimination, substitution and graphical methods. Appropriate steps were taken to solve the equations respectively.

➤ **Assignment**

- Solve by elimination method $3t - 4s = 5$, $2t + 3s = 7$

- Solve by simultaneous method $3x - 2y = 1$, $2x + y = 5$
- Solve graphically $x - y = 3$, $2x - y = 4$. Take values of x from -3 to 3 and the scale of 2cm to 2 unit on y axis and 1cm 1 unit on x axis.

➤ *Lesson Plan for Statistics*

Subject: Basic General Mathematics

Class: Year 2

Topic: Statistics

Duration: 1 hour

Course Code: GSE 222

➤ *Instructional Objectives*

At the end of the lesson the students should be able to: collect, tabulate and represent data.(2) make a frequency distribution table.(3) make graphical representation of data with ease and 80% success.

➤ *Entry Behaviour*

The students have known: (1) the four basic operation on numbers (2) making of table of values and plotting of graph.

➤ *Instructional Materials*

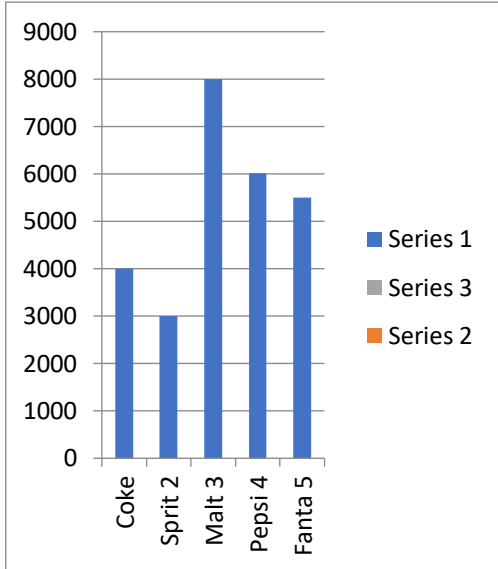
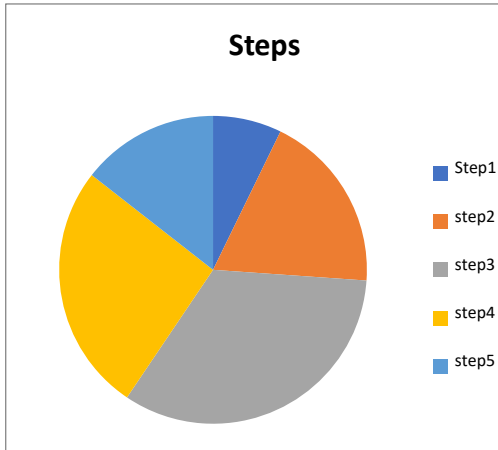
Class register, score sheets, and students matriculation list are use as example of data, while Graph board and graph book are used to illustrate graphical representation of data.

➤ *Content Development*

• *Introduction:*

Data are any numerical facts or information which can be measured or given a numerical qualification. There are two types of data mostly used in statistics these are: (1) discrete data:- the data obtained by counting the number of things e.g number of students in the class. Discrete data do not take values in between two numbers. (2) continuous data:- these are data obtained by measuring things e.g weight, height, students performance in mathematics test and so on. Continuous data takes values in between numbers. When data are collected, they are expected to be analysed, summarised and interpreted before they can be used for whatever for which they are collected. Apart from frequency distribution table, a clearer method of presenting data is the graphical method.

Content Development	Activities					Skills Emphasized	
	Teacher			Students			
Step I: Tabular presentation of data <i>The number of samples in an observation may be so large that before any tabulations are made, the raw data must be arranged in a suitable form.</i>	<i>Before the experiment, students are given a list of 70 students' marks in a test. The marks are: 71, 68, 45, 55, 59, 56, 66, 64, 52, 72, 51, 67, 62, 64, 70, 57, 59, 73, 54, 51, 59, 65, 67, 62, 64, 45, 59, 59, 63, 59, 44, 72, 67, 54, 66, 64, 62, 70, 59, 55. Additive Frequency and tally table. The class mark is 47 and the percent mark is 73. Mark of the class is 74-47=27=3 approximately. Therefore each group should contain 4 units in order to have several equal class intervals.</i>					They solve along with the lecturer and solve 1 and 2 in their notebook following the lecturer's example.	Set induction and use of examples
	Class interval	Tally	Frequency				
	47-50	—1 1	7				
	51-54	—1 1	7				
	55-58	—1 1	7				
	59-62	—1 1 1	8				
	63-64	—1 1 1 1 1	1	1			
	67-70	—1 1	6				
	71-74	1 1 1 1	4				
	Class interval	Class limit	Class boundary	Class mark	Cumulative Frequency		
47-50	47,50	46.5,50.5	48.5		7		
51-54	51,54	50.5,54.5	52.5	1			
55-58	55,58	54.5,58.5	56.5	2			
59-62	59,62	58.5,62.5	60.5	2			
63-66	63,66	62.5,66.5	64.5	4			
67-70	67,70	66.5,70.5	68.5	4			

	71-74	71,74	70.5,74.5	72.5	5	0																																																																	
<div>Step II: Graphical presentation of Data.</div> <div>The graphical method of presenting data is known as graphical representation. The graphical representation can be represented in the form of bar chart and Pie chart.</div>	<div>6.1 The amount of the head of school to be paid to the teachers is given as follows: Class-4th, 2000; 5th, 3000; 6th, 4000; 7th, 5000; 8th, 6000. What is the total amount of the head of school to be paid to the teachers?</div> <div>2) The salary schedule for Lecturers in university of Calabar has five steps. Here are the salaries and number of Lecturers for each step.</div> <table><thead><tr><th>Step</th><th>Salary</th><th>Lecturers</th></tr></thead><tbody><tr><td>1</td><td>₦25,000</td><td>42</td></tr><tr><td>2</td><td>₦28,000</td><td>98</td></tr><tr><td>3</td><td>₦35,000</td><td>138</td></tr><tr><td>4</td><td>₦45,000</td><td>8</td></tr><tr><td>5</td><td>₦55,000</td><td>3</td></tr></tbody></table> <div>Construct a pie chart that represent this information. (b) What is the mean salary in the university? (c) A Lecturer's contribution to the school district's retirement plan is ₦500 plus 5% of his or her salary. What is the mean contribution?</div> <div></div> <div>2) Solution: (a) To construct the pie chart multiply the amount by the number of lecturer in each of the steps to have;</div> <table><tbody><tr><td>₦</td><td>1</td><td>,</td><td>0</td><td>5</td><td>0</td><td>,</td><td>0</td><td>0</td></tr><tr><td>₦</td><td>2</td><td>,</td><td>7</td><td>4</td><td>4</td><td>,</td><td>0</td><td>0</td></tr><tr><td>₦</td><td>4</td><td>,</td><td>8</td><td>3</td><td>0</td><td>,</td><td>0</td><td>0</td></tr><tr><td>₦</td><td>3</td><td>,</td><td>7</td><td>8</td><td>0</td><td>,</td><td>0</td><td>0</td></tr><tr><td>₦</td><td>2</td><td>,</td><td>0</td><td>9</td><td>0</td><td>,</td><td>0</td><td>0</td></tr></tbody></table> <div>Total number of staff is 400. Divide each of the values in each step by 400 and multiply by 360° convert to the nearest degree to have 36°, 69°, 120°, 34° and 59°. Use this to construct the pie chart.</div> <div></div> <div>The mean salary in the university is $\frac{₦14,494,000}{400} = ₦36,235$</div> <div>(c) The mean contribution will be $₦36,235 \times 0.5 + ₦500 = ₦2,311.75$.</div>						Step	Salary	Lecturers	1	₦25,000	42	2	₦28,000	98	3	₦35,000	138	4	₦45,000	8	5	₦55,000	3	₦	1	,	0	5	0	,	0	0	₦	2	,	7	4	4	,	0	0	₦	4	,	8	3	0	,	0	0	₦	3	,	7	8	0	,	0	0	₦	2	,	0	9	0	,	0	0	<div>Participate in</div> <div>Plotting the graph of the equations into their graph books.</div>	<div>Explanation and use of examples</div>
Step	Salary	Lecturers																																																																					
1	₦25,000	42																																																																					
2	₦28,000	98																																																																					
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5	₦55,000	3																																																																					
₦	1	,	0	5	0	,	0	0																																																															
₦	2	,	7	4	4	,	0	0																																																															
₦	4	,	8	3	0	,	0	0																																																															
₦	3	,	7	8	0	,	0	0																																																															
₦	2	,	0	9	0	,	0	0																																																															
<div>E v a l u a t i o n</div>	<div>In the biological sciences, the use of graphical representation is very important. It helps to present data in a clear and concise manner. It also helps to identify trends and patterns in the data. In the biological sciences, the use of graphical representation is very important. It helps to present data in a clear and concise manner. It also helps to identify trends and patterns in the data.</div>						<div>Solve the exercise and assignment given by the teacher in their note book</div>	<div>Use of examples, planned repetition and</div>																																																															

➤ *Summary*

In our lesson today, we learnt that data are any numerical facts or information which can be measured or given a numerical qualification. That there are two types of data mostly used in statistics these are: discrete data and continuous data. That the number of sample in an observation may be so large that before any deductions are made about them, the raw data must be arranged in a tabular form. That apart from frequency distribution table, a clearer method of presenting data is the graphical method.

➤ *Assignment*

In a college Mathematics class all the students are also taking anthropology, history, or psychology and some of the students are taking two or even all three of these courses. If (i) forty students are taking anthropology, (ii) eleven students are taking history, (iii) twelve students are taking psychology, (iv) three students are taking all three courses, (v) six students are taking anthropology and history, and (vi) six students are taking psychology and anthropology. Represent the information on a pie chart.

- How many students are taking only anthropology?
- How many students are taking anthropology or history?
- How many students are taking history and anthropology, but not psychology?

APPENDIX E IV**➤ Teaching Plan For Writing To Learn Group****Subject:** Basic General Mathematics IV**Class:** year 2**Topic :** Variation**Duration:** 1 Hour**Course Code:** GSE 222**➤ Instructional Objectives**

At the end of the lesson the students should be able to:-

- Solve correctly with ease and 75% success some problem involving direct variation.
- Solve with ease and 75% success some problem on inverse variation.
- Solve with ease and 75% success some problem on joint variation.
- Solve correctly with ease and 75% success some problem on partial variation.

➤ Entry Behaviour

The students have known:-

- Solving Equation Involving Fractions.
- Change of Subject of Formular.
- Solving Simultaneous Equation in Two Variables.

➤ Instructional Material

A chart showing solved problems on variations and reciprocal table.

➤ Content Development• **Introduction:**

The word variation is used to describe the relationship which exist among two quantities. In everyday life we come across relationships between quantities. These relationship can be seen in terms of varying one quantity to another. For instance if a worker is paid a certain rate for every hour that he works, the total wages due to him in a given period will depend on the hours he works. Hence a kind of variation exist.

Content development	A c t i v i t i e s		Writing to learn and Reciprocity Activities	Strategies and Skills Emphasized
	T e a c h e r s	S t u d e n t s		
Step I: Direct variation When the value of one variable is constant, then the value of the other variable varies directly with it. Example: The price of a book is \$5.00. How much will I have to pay for 10 books? Solution: The price of a book is \$5.00. So, the value of the other variable varies directly with it. Form the variation equation using the given variables, $C = KL + 50$. Solve for C algebraically or graphically, elimination or substitution to have \$65.00.		This is a case of direct variation. The value of the other variable varies directly with it. The constant of variation, k, is 6. $12 \times 6 = 72$. Since 4 are absent, it leaves 8 workers (12-4=8). $3Y = 72x$, $y = 72/x$, $x = 8$ therefore $y = 72/8 = 9$ hrs.	They choose a price. They write the problem. They solve the problem and check the answer. They write the answer. They write the solution. They write the conclusion. They write the reflection.	The writing to learn and reciprocity strategies that is emphasized are Cognitive Academic Learning Style.
Step II: Inverse variation When the value of one variable is constant, then the value of the other variable varies inversely with it. Example: A car travels 120 km in 2 hours. How long will it take to travel 180 km? Solution: A car travels 120 km in 2 hours. So, the value of the other variable varies inversely with it. Form the variation equation using the given variables, $C = KL + 50$. Solve for C algebraically or graphically, elimination or substitution to have \$65.00.	(c) How does what we learned today apply in real life? The lecturer guide the student on how to solve the problem.	This is a case of inverse variation. The value of the other variable varies inversely with it. The constant of variation, k, is 6. $12 \times 6 = 72$. Since 4 are absent, it leaves 8 workers (12-4=8). $3Y = 72x$, $y = 72/x$, $x = 8$ therefore $y = 72/8 = 9$ hrs.	Lecturer guide the student on how to solve the problem. They write the problem. They solve the problem and check the answer. They write the answer. They write the solution. They write the conclusion. They write the reflection.	Writing To Learn Mathematics Strategy :- Concept Definition Map Reciprocity (Reflection) Reciprocity representation is which the concept, writing terms and example are related to a main topic.

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Content Development	A c t i v i t i e s	
	T e a c h e r s	S t u d e n t s
Step I: Lowest common multiples of algebraic <i>The lowest common multiple (number) of 6 and 4 means a lowest number which is not 4 or 6, which can also be divided by 6 and 4 without any remainder. We can find 4 = 4, 8, 12, 16, 20, 24. It can be seen that amongst the common multiple that 12 is the least.</i>	Ex. I: Find the L.C.M of the number (1) 9, 6 and 3 (2) x^2 , y and Zy. S o l u t i o n (1) $9 = 3 \times 3$ $6 = 3 \times 2$ $3 = 3 \times 1$ $L.C.M = 3 \times 3 \times 2 = 18$ (2) $x^2 = x \times x \times x$ $y = 1 \times y$ $z y = z \times y$ $L . C . M = x \times x \times x \times y \times z$ $= x^3 y z$	
STEP II: Simple Equations involving brackets <i>Simple equation involving brackets can be solved by first removing the bracket.</i>	<i>Q2. Mr. Jeeves earned a check and the bank told him that he had to cash it for money. He had taken out a check for \$100 and \$50. He had \$100 in his pocket and \$50 in his bank account. He had \$100 in his pocket and \$50 in his bank account. He had \$100 in his pocket and \$50 in his bank account.</i> 2 . S o l v e $4 (3 x - 2) = 6$ <i>Solution: Let x = number of dollars, y = number of cents. The problem states that $100x + 50y = 20000 + 50$, which simplifies to $100x + 50y = 20050$. The first observation gives us that $x = 20$ and $y = 1$. Solving simultaneously, we get $x = 20$, $y = 1$.</i> $2 . 4 (3 x - 2) = 6$ $4 x 3 x - 4 x 2 = 6$ $1 2 x - 8 = 6$ A d d 8 t o b o t h s i d e s $1 2 x + 8 = 6 + 8$ $1 2 x = 1 4$ D i v i d e s b o t h s i d e s b y 1 2 $\frac{12x}{12} = \frac{14}{12}$ $x = \frac{14}{12}$	<i>Method 1: This is the case of solution of two variable linear equations, writing and solving linear equations given real life situations.</i> <i>As first glance, this seems like a lot of guessing and checking. With 100 dimes for dollars and 100 cents for cents, I would have to check out 10,000 possibilities. With making guessing it was becomes apparent that the</i> <i>Method 2: Let x = number of dollars, y = number of cents. The problem states that $100x + 50y = 20000 + 50$, which simplifies to $100x + 50y = 20050$. The first observation in method 1 gives us that $x = 20$ and $y = 1$. Solving simultaneously,</i> $2) 1 2 x + 8 = 6 + 8$ $1 2 x = 1 4$ <i>D i v i d e s b o t h s i d e s b y 1 2</i> $\frac{12x}{12} = \frac{14}{12}$ $x = \frac{14}{12}$

<p>STEP III: Simple Equation Involving Algebraic Fraction</p> <p><i>A simple equation involving algebraic fraction can be solved as ordinary simple equation except that L.C.M should be written multiply throughout the equation, if it is in which</i></p>	<p>Ex. III: Solve the following (1) $\frac{3}{4x} + \frac{1}{2} = 1$. A store costs \$20. You make \$50 down payment and monthly payments of \$42. How many months will it take to finish paying for the store? Give explanation to your answer.</p> <p>S o l u t i o n</p> <p>2. $\frac{3}{4x} + \frac{1}{2} = 1$ S u b t r a c t $\frac{1}{2}$ from both sides</p> $\frac{3}{4x} + \frac{1}{2} - \frac{1}{2} = 1 - \frac{1}{2}$ $\frac{3}{4x} = \frac{1}{2}$ <p>M u l t i p l y t h r o u g h b y 4</p> $\frac{3}{4x} \times 4 = \frac{1}{2} \times 4$ $\frac{3x}{x} = 2$ <p>D i v i d e t h r o u g h b y 3</p> $\frac{3x}{3} = \frac{2}{3}$ $x = \frac{2}{3}$	<p>2. SIMBA'S NE-SPRING has a store costs \$20, you make \$50 and will pay \$42 per month. Let x = the number of months to pay. You make use of the store and what the number represented. What is the number?</p>
<p>E v a l u a t i o n</p>	<p>Evaluates the students by given them some problems on algebraic expression as exercise and assignment to solve in their note book.</p>	<p>Ex. III: Solve the following (1) $\frac{3}{4x} + \frac{1}{2} = 1$</p> <p>S o l u t i o n</p> <p>3. $\frac{3}{4x} + \frac{1}{2} = 1$ S u b t r a c t $\frac{1}{2}$ from both sides</p> $\frac{3}{4x} + \frac{1}{2} - \frac{1}{2} = 1 - \frac{1}{2}$ $\frac{3}{4x} = \frac{1}{2}$ <p>M u l t i p l y t h r o u g h b y 4</p> $\frac{3}{4x} \times 4 = \frac{1}{2} \times 4$ $\frac{3x}{x} = 2$ <p>D i v i d e t h r o u g h b y 3</p> $\frac{3x}{3} = \frac{2}{3}$

		$x = \frac{3}{3}$
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➤ *Summary:*

In our lesson we have learnt that an equation is a mathematical statement that shows equality between two expressions. That a linear equation is a mathematical statement that contains only term in one variable and constant, and the highest power of the variable is 1. Also that a simple equation involving algebraic fraction can be solved as ordinary simple equation except that L.C.M should be used to multiply through to clear the fraction, if so which.

• *Assignment: (1)*

Aman is four times as old as his son. In four years time he will be three times as old. What are their ages now?

The same number is added to both the numerator and the denominator of the fraction $\frac{7}{17}$. If the fraction then becomes $\frac{6}{11}$, find the number added.

➤ *Week Two*

Lesson Plan For Quadratic Equation With Writing to learn and Reciprocity methods

Subject: Basic General Mathematics IV

Class: year 2

Topic : Quadratic Equation

Duration: 1 Hour

Course Code: GSE 222

➤ *Instructional Objectives*

- At the end of the lesson, the students should be able to: Solve correctly quadratic equation by factorization method with ease and 85% success.
- Solve with ease and 70% success quadratic equation of the type $(ax + b) = c$ where a, b, c are real numbers and x a variable.
- Solve quadratic equation with imaginary roots with ease and 75% success.
- Solve correctly with ease and 75% success some problem on quadratic equation by using quadratic formula.

➤ *Entry Behaviour*

The students have known: (1) Factorization of quadratic expression (2) simplification of quadratic expression.

➤ *Instructional Material*

A flip chart showing the steps to be taken when solving quadratic by completing squares.

➤ *Content Development*

• *Introduction:*

A quadratic equation is an equation of second degree, that is an equation in which 2 is the highest power of the unknown .

Content development	A c t i v i t i e s		Writing to learn and Reciprocity Activities	Strategies and Skills Emphasized
	L e c t u r e r s	S t u d e n t s		
<p>STEP 1: Solving Quadratic Equation By Factorization</p> <p>Factorization of quadratic expressions simply means splitting up its number or digits</p>	<p>Ex II: Suppose the area of a rectangular field is $x^2 + 11x + 18$ and the length of one of its sides is $x + 2$. Find the other side.</p> <p>Factorize: $(2) 4x^2 + 13x + 9(3) x^2 + 11x + 18 = 0$.</p> <p>S o l u t i o n s</p> <p>(3) The factors are -3 and -6 and 2 and 3. $\therefore (4x^2 + 13x + 9) = (4x + 9)(x + 2)$</p> <p>(4) $4x^2 + 13x + 9$ the product of 4 and 9 is 36. Factors that will give the product 36 are 4 and 9. $\therefore 4x^2 + 13x + 9 = (4x + 9)(x + 2)$</p>	<p>They solve along with the lecturer and solve 3 and 4 on their own with following statement example.</p> <p>3. Solve this problem is similar to quadratic equation. Try factoring as a solution method. The unknown quantity is x.</p> <p>4. I know a quadratic like the one above. Here, I have to solve the problem is to factor. I assigned that x^2 is a variable.</p>	<p>Activates student's background knowledge. Helps students monitor their comprehension of text. Stimulates questioning and analysis of text. Helps students</p>	<p>Writing-To-Learn: Mathematics Strategy 8: Marginal Notes and Reciprocity (Connecting).</p> <p>Marginal notes and connecting as short written statements in which students record their interactions with the text in the margins while they are solving problems. The connecting integrates reading, writing, and</p>
<p>STEP II: Solving Quadratic Equation of the Type $ax^2 + bx + c = 0$ and connection with</p> <p>$4x^2 - 12x + 9 = 0$. This is the same as $(4x - 6)^2 = 0$ and $4x - 6 = 0$. We will</p>	<p>Ex. III: 1. Solve the equation: $4(2-3)^2 - 36 = 0$.</p> <p>$4(2-3)^2 - 36 = 0$ $4(2-3)^2 = 36$ $(2-3)^2 = 9$ $2-3 = \pm 3$ $2-3 = 3$ or $2-3 = -3$ $2 = 3+3$ or $2 = -3+3$ $2 = 6$ or $2 = 0$</p> <p>S o l u t i o n</p> <p>4. The following is an example of an algebraic equation: $4(2-3)^2 - 36 = 0$ $4(2-3)^2 = 36$ $(2-3)^2 = 9$ $2-3 = \pm 3$ $2-3 = 3$ or $2-3 = -3$ $2 = 3+3$ or $2 = -3+3$ $2 = 6$ or $2 = 0$</p>	<p>Listen and participate in solving along with the lecturer.</p> <p>They expand the given expressions in their note book.</p> <p>2. $(x - 1)^2 = 4$ Take square root of both sides $x - 1 = \pm \sqrt{4}$ $x - 1 = +2$ or $x - 1 = -2$ $\therefore x - 1 = +2$ or $x - 1 = -2$ $x = 1 + 2$ or $x = 1 - 2$ i.e. $x = 3$ or $x = -1$</p>		

		$= -2 + 1$ $\therefore x = 3 \text{ or } -1$ <p>5. $(2x - 1)^2 =$</p> <p>Take square root of both sides</p> $2x - 1 = \sqrt{3} \text{ add 1 to both sides}$ $2x = 1 + \sqrt{3} \text{ divide through by 2}$ $x = \frac{1}{2} + \frac{\sqrt{3}}{2}$ <p>i.e. either $x = \frac{1}{2} + \frac{\sqrt{3}}{2}$ or $x = \frac{1}{2} - \frac{\sqrt{3}}{2}$, $\sqrt{3} = 1.73$</p> $\text{So } x = \frac{1}{2} + 1.73 \text{ or } \frac{1}{2} - 1.73 = 0.5 + 1.73 \text{ or } 0.5 - 1.73$ $\therefore x = 1.23 \text{ or } 0.5 - 1.73 = -1.23$ $\underline{1 + 1.73}$		
<p>STEP III: Graph of Quadratic Equation</p> <p>Another way of solving quadratic equation is by graphical method. If 'y' is related to 'x', then the set of values of 'x' is called domain which contains a set of real numbers and vice versa.</p>	<p>Ex. I: Solve the quadratic equations:</p> <p>(1) $x^2 + 8x = 0$</p> <p>(2) $3t^2 - 75t = 0$</p> <p>S o l u t i o n</p> <p>4. $x^2 + 8x = 0$ factorize L</p> $x(x + 8) = 0$ $x = 0 \text{ or } x + 8 = 0$ $x = -8$ $\therefore x = 0 \text{ or } -8$	<p>Factorize the algebraic expressions on the board.</p> <p>To solve quadratic graphically the following steps are considered:</p> <ol style="list-style-type: none"> 1) Make a table of values for the function which contains the values of 'x' and 'y'. 2) Choose a suitable scale for both axes of the graph. 3) Plot the point on the graph. 4) Obtain the roots of the equation which is the intersection of the curve with the x-axis. 	<p>Solutes are provided a visual prompt and asked to explain with a description or explanation. The prompt can be a quote, demonstration, photograph, or video.</p> <p>Enhance the appearance of your construction using the stylebar.</p>	<p>Writing-To-Learn: Mathematics Strategy 9: Quick Write and Reciprocity Solving</p> <p>Quick writing asks students to independently spend everything they can think of in 5 minutes. The prompt students are given can be open-ended or specific, depending on the lecturer's purpose. This strategy is used to help students think about a problem or concept. It provides information for the lecturer on the student's understanding, i.e. formative assessment. It helps students monitor their own learning. It allows students to share their ideas with the class. Lecturers are provided with a number of problem-solving options, though not directed to a specific problem-solving strategy.</p>
Evaluation	<p>4) Factorize $v^2 - 6v - 16$</p> <p>5) Factorize $36 - 12t - 16t^2$</p> <p>6) Solve using formula $2x^2 - 4x - 4 = 0$</p>	Solve the exercise and assignment given by the lecturer in their note book		C l o s u r e

➤ Summary

In our lesson, we have studied more about solving quadratic equation, by the formula and factor method. We also looked into Imaginary root of quadratic equations and the solution to quadratic equation such as $(ax + b)^2 = c$.

➤ *Assignment*

- Factorize $x^2 - 10x + 39$, explain steps taken to solve the problem.
- Solve the equation (a) $(2x + 1)^2 = 5$. Give explanation.
- Solve using the formula $2x^2 + 15x + 7 = 0$. Was the formula method very easy for you?
- You and your friend are having trouble graphing quadratic functions of the form $y = ax^2$ and $y = ax^2 + c$. Your friend asks you to write some generalizations to help her graph these types of equations. (a) Explain the role of a. (b) Explain the maximum and minimum. (c) What is the vertex? (d) Explain the role of c.
- Suppose $2-6+=0$ and $2-6+=2$ are equivalent to $(-)^2=7$ and $(-)^2=$, respectively. Find the value of . Explain the steps used in solving the problem to support your answer. b) Identify the Mathematical concept(s) that you used to solve the problem and define each one.

➤ *Lesson Plan for Simultaneous Linear Equations in Two Variables***Subject:** Basic General Mathematics IV**Class** Year 2**Topic** Solution of Simultaneous Linear equations in two variables**Duration:** 1hour**Course Code:** GSE 222➤ *Instructional Objectives*

At the end of the lesson, the students should be able to solve simultaneous linear equation by (1) elimination method (2) substitution method (3) graphical method with ease and 80% success.

➤ *Entry Behaviour*

The students have known (1) linear equation in one variable and two variables (2) quadratic equation in one variable.

➤ *Instructional Material*

Chart of solved simultaneous equation by elimination, substitution and graphical methods.

➤ *Content Development*• *Introduction:*

A linear equation is the equality of two linear expressions and a constant. It is solved simultaneously to obtain the two variables. There are methods of solving these equations which are elimination, substitution and graphical methods.

Content development	A c t i v i t i e s		Writing to learn and Reciprocity Activities	Strategies and Skills Emphasized
	T e a c h e r s	S t u d e n t s		
STEP I: Elimination Method <small>Working from these equations, elimination method is used to solve simultaneous linear equations by using the elimination method.</small>	<p>Ex. I: Using elimination method solve the following equation.</p> <p>(1) The height of a tuber of yam and 2 goats is ₦220 and the height of 15 tubers of yam and 2 goats is ₦170. Find out the price of tuber of yam and goats.</p> <p>(2) The height of a tuber of yam and 2 goats is ₦220 and the height of 15 tubers of yam and 2 goats is ₦170. Find out the price of tuber of yam and goats.</p> <p>Solution: (1) Let the price of tuber of yam and goats be x and y respectively. Then the equations are:</p> <p>15 tuber of yam and 2 goats will be $(15x + 2y) = ₦170$</p> <p>$5x + 3y = ₦220$.....i</p> <p>$15x + 2y = ₦170$.....ii multiply (i) by 3, (ii) by 1</p> <p>$15x + 9y = ₦660$.....iii</p> <p>$15x + 2y = ₦170$.....iv</p> <p>Subtract iv from iii</p> <p>$7y = 490$</p> <p>$Y = 70$, put in (i)</p> <p>$5x + 3(70) = ₦220$</p> <p>$5x + 210 = 220$</p> <p>$5x = 10$</p> <p>$X = 2$, yam = ₦2 and goat = ₦70.</p> <p><small>Solution: (2) Let the price of tuber of yam and goats be x and y respectively. Then the equations are:</small></p> <p>15 tuber of yam and 2 goats will be $(15x + 2y) = ₦170$</p> <p>$5x + 3y = ₦220$.....i</p> <p>$15x + 2y = ₦170$.....ii multiply (i) by 3, (ii) by 1</p> <p>$15x + 9y = ₦660$.....iii</p> <p>$15x + 2y = ₦170$.....iv</p> <p>Subtract iv from iii</p> <p>$7y = 490$</p> <p>$Y = 70$, put in (i)</p> <p>$5x + 3(70) = ₦220$</p> <p>$5x + 210 = 220$</p> <p>$5x = 10$</p> <p>$X = 2$, yam = ₦2 and goat = ₦70.</p>	<p>They solve along with the lecturer and solve 1 and 2 in their note book following the teacher's example.</p> <p><small>After solving the problem, the students are asked to write a summary of the steps taken to solve the problem. A problem-solving chart is also prepared.</small></p>	<p>Students are asked to write a summary of the steps taken to solve the problem. A problem-solving chart is also prepared.</p>	<p>Writing-To-Learn: Mathematics Strategy 15: Argumentation and Reciprocity (Reflection)</p> <p>The writing-to-learn strategy is used to help students develop their understanding of the concept of simultaneous linear equations. The strategy involves students writing down their thoughts and feelings about the problem, and then sharing their work with a partner or the class. This helps students to develop their communication skills and to learn from each other.</p>

STEP II: Substitution Method <small>Substitution method used to solve a system of two linear equations in two variables. It involves solving one equation for one variable and substituting the result into the other equation to solve for the other variable.</small>	Ex. III: What is the length of the side of a rectangle when the perimeter is 32 cm and area 55 cm ² ? 2. Solve by substitution method the following (1) $x - 2y = 4$, $2x + 5y = 17$ (2) $4x - 3y = 11$, $2x + 5y = 25$. <small>Solution: Let the length of rectangle be x and width be y then perimeter is $2(x+y)=32$ and area is $xy=55$. Solving these equations we get x=11 and y=5.</small>	L i s t e n a n d participate in solving along with the teacher. They expand the g i v e n expressions in their note book.	Create Slider m-1 with the default settings for sliders. Create Slider b-1 with the default settings for sliders. Create the linear equation line 1 Create slider m-2 using the default settings for slider Create linear equation line 2. Create the dynamic text 1. Create the dynamic text 2.	E x p l a n a t i o n a n d u s e o f e x a m p l e s
STEP III: Graphical Method <small>To solve simultaneous equations graphically, represent the variables in the equations as lines on a Cartesian plane. The intersection point of the lines represents the solution to the system of equations.</small> Step IV: Graphical Solution of Simultaneous Linear Equations and Quadratic Equations <small>To solve simultaneous equations graphically, represent the variables in the equations as lines or parabolas on a Cartesian plane. The intersection point of the lines or parabolas represents the solution to the system of equations.</small>	Ex. II: Solve the simultaneous equations $4x + 3y = 12$ and $2x + 5y = 25$ by graphical method. Ex. I: What is the values of x and y in the equation (1) $3x + 2y = 12$, $xy + 5y = 21$ (2) $3x + y^2 = 22$, $2x - y = 10$ (3) $3x + 4y = 11$, $xy = 2$ <small>Solution: We have to solve the equations graphically. For (1), we have $3x + 2y = 12$ and $xy + 5y = 21$. Solving these equations we get x=2 and y=3. For (2), we have $3x + y^2 = 22$ and $2x - y = 10$. Solving these equations we get x=4 and y=-2. For (3), we have $3x + 4y = 11$ and $xy = 2$. Solving these equations we get x=1 and y=2.</small>	F a c t o r i z e t h e a l g e b r a i c e x p r e s s i o n s l o n t h e b o a r d . Ex. I: What is the values of x and y in the equation (1) $3x + 2y = 12$, $xy + 5y = 21$ (2) $3x + y^2 = 22$, $2x - y = 10$ (3) $3x + 4y = 11$, $xy = 2$ <small>Solution: We have to solve the equations graphically. For (1), we have $3x + 2y = 12$ and $xy + 5y = 21$. Solving these equations we get x=2 and y=3. For (2), we have $3x + y^2 = 22$ and $2x - y = 10$. Solving these equations we get x=4 and y=-2. For (3), we have $3x + 4y = 11$ and $xy = 2$. Solving these equations we get x=1 and y=2.</small>	Construct the intersection point of both lines, line 1 and line 2. Define x coordinate = x (A). Define y coordinate = y(A). Create the dynamic text 3. Create the dynamic text 4. Find the text and sliders so they can't be moved accidentally.	E x p l a n a t i o n a n d s t i m u l u s v a r i a t i o n
E v a l u a t i o n		Solve the exercise and assignment given by the teacher in their note book		U s e o f e x a m p l e s , p l a n n e d r e p e t i t i o n a n d C l o s u r e
	Solve the simultaneous equation by elimination method $4x + 3y = 12$, $2x + 5y = 25$. Solve the simultaneous equation by substitution $x - 2y = 4$, $2x + 5y = 25$. Solve graphically $2x + y = 3$, $x + y = 7$. Take values of x from -4 to 4, use the scale of 1 cm to represent 1 unit.			

➤ Summary

In our lesson, we have learnt that simultaneous equations in two variables can be solved by elimination, substitution and graphical methods. Appropriate steps were taken to solve the equations respectively.

➤ Assignment

- Suppose $2 - 6 + 0$ and $2 - 6 + 2$ are equivalent to $(-)^2 = 7$ and $(-)^2 =$, respectively. Find the value of . Explain the steps used in solving the problem to support your answer. b) Identify the Mathematical concept(s) that you used to solve the problem and define each one.
- Write a paragraph (9-10 complete sentences) explaining each of the three ways to solve systems of equations. The three ways are: graphing, substitution, and elimination. Use your book and notes to help you. In your paragraph state which method you prefer and why. Then select one of the three problems below and solve it using your preferred method. Bonus: Solve the other two problems using each of the other two methods.

$$y = 3x + 5 \quad 2. 9x + 4y = -17 \quad 3. 5x - 7y = -21$$

$$2y - 6x = 4 \quad 12y = -3x - 3x \quad 14y - 5y = 22$$

➤ Lesson Plan for Statistics

Subject: Basic General Mathematics

Class: Year 2

Topic: Statistics

Duration: 1 hour

➤ **Instructional Objectives**

At the end of the lesson the students should be able to: collect, tabulate and represent data.(2) make a frequency distribution table.(3) make graphical representation of data with ease and 80% success.

➤ **Entry Behavior**

The students have known: (1) the four basic operation on numbers (2) making of table of values and plotting of graph.

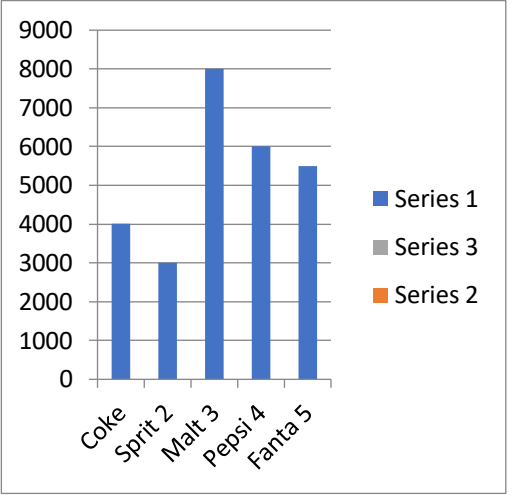
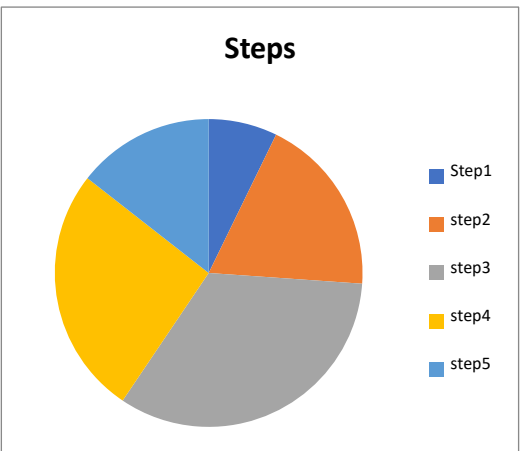
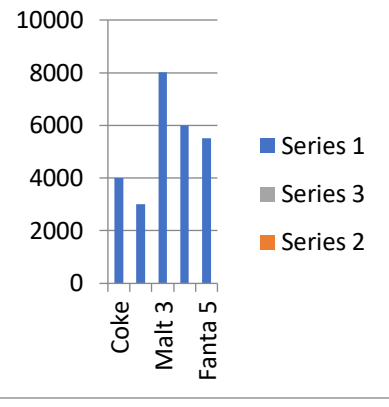
➤ **Instructional Materials**

Class register, score sheets, and students matriculation list are use as example of data, while Graph board and graph book are used to illustrate graphical representation of data.

➤ **Content Development**• **Introduction:**

Data are any numerical facts or information which can be measured or given a numerical qualification. There are two types of data mostly used in statistics these are: (1) discrete data:- the data obtained by counting the number of things e.g number of students in the class. Discrete data do not take values in between two numbers. (2) continuous data:- these are data obtained by measuring things e.g weight, height, students performance in mathematics test and so on. Continuous data takes values in between numbers. When data are collected, they are expected to be analysed, summarised and interpreted before they can be used for whatever for which they are collected. Apart from frequency distribution table, a clearer method of presenting data is the graphical method.

Content development	A	c	t	i	v	i	t	i	e	s																																																																			
	T	e	a	c	h	e	r	s	S	t	u	d	e	n	t	s																																																													
Step 1: Thinking presentation of data The number of supplies in a warehouse are as follows:	<p>Example 1: In the warehouse examination for various supplies are as follows: 67 70 66 55 59 66 66 64 55 72 55 67 62 66 70 55 56 59 65 58 64 72 67 61 66 66 52 66 72 55 55 66 65 59 66 66</p> <p>(i) To show how to collect and represent data of individual only as representative of the warehouse supplies. To make a frequency distribution table of the data. (ii) To show the highest brand of soft drink consumed. To make a bar chart and realize that the data has been collected, this means that the data is the highest consumed brand. (iii) To make the frequency distribution table of the data by using the data collected and find the frequency of the data. (iv) The average soft drink consumption in a week is calculated as $4000 + 3000 + 8000 + 6000 + 5500 = 26,500 \div 5 = 5,300$ bottles. (v) The average number of supplies in a warehouse is calculated as $4000 + 3000 + 8000 + 6000 + 5500 = 26,500 \div 5 = 5,300$ bottles. The least mark is 47 and the greatest mark is 74. Width of the class is $74 - 47 = 27 \div 4 = 6.75 \approx 7$. Therefore, each group should contain 7 units in order to have even groups of class intervals.</p> <table><tr><th>Class interval</th><th>T a l l y</th><th>Frequency</th></tr><tr><td>47-50</td><td> —1 1</td><td>7</td></tr><tr><td>51-54</td><td> —1 1</td><td>7</td></tr><tr><td>55-58</td><td> —1 1</td><td>7</td></tr><tr><td>59-62</td><td> —1 1 1</td><td>8</td></tr><tr><td>63-64</td><td> —1 1 1 1</td><td>1</td></tr><tr><td>67-70</td><td> —1</td><td>6</td></tr><tr><td>71-74</td><td>1 1 1 1</td><td>4</td></tr></table> <p>Solution: (1) Frequency and tally table.</p> <p>The least mark is 47 and the greatest mark is 74. Width of the class is $74 - 47 = 27 \div 4 = 6.75 \approx 7$. Therefore, each group should contain 7 units in order to have even groups of class intervals.</p> <table><tr><th>Class interval</th><th>Class limit</th><th>Class boundary</th><th>Class mark</th><th>Cum Freq</th></tr><tr><td>47-50</td><td>47,50</td><td>46.5,50.5</td><td>48.5</td><td>7</td></tr><tr><td>51-54</td><td>51,54</td><td>50.5,54.5</td><td>52.5</td><td>1 4</td></tr><tr><td>55-58</td><td>55,58</td><td>54.5,58.5</td><td>56.5</td><td>2 1</td></tr><tr><td>59-62</td><td>59,62</td><td>58.5,62.5</td><td>60.5</td><td>2 9</td></tr><tr><td>63-66</td><td>63,66</td><td>62.5,66.5</td><td>64.5</td><td>4 0</td></tr><tr><td>67-70</td><td>67,70</td><td>66.5,70.5</td><td>68.5</td><td>4 6</td></tr><tr><td>71-74</td><td>71,74</td><td>70.5,74.5</td><td>72.5</td><td>5 0</td></tr></table>											Class interval	T a l l y	Frequency	47-50	—1 1	7	51-54	—1 1	7	55-58	—1 1	7	59-62	—1 1 1	8	63-64	—1 1 1 1	1	67-70	—1	6	71-74	1 1 1 1	4	Class interval	Class limit	Class boundary	Class mark	Cum Freq	47-50	47,50	46.5,50.5	48.5	7	51-54	51,54	50.5,54.5	52.5	1 4	55-58	55,58	54.5,58.5	56.5	2 1	59-62	59,62	58.5,62.5	60.5	2 9	63-66	63,66	62.5,66.5	64.5	4 0	67-70	67,70	66.5,70.5	68.5	4 6	71-74	71,74	70.5,74.5	72.5	5 0	Writing to learn and Readiness: Activities	Strategy and Skills Emphasized
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	<p>1 #25,000 42 2 #28,000 98 3 #35,000 138 4 #45,000 84 # 5 5 , 0 0 0</p>  	<p>s o l v i n g a l o n g w i t h t h e t e a c h e r . They expand the given expressions in their note book. S o l u t i o n : (1)</p> 		
Evaluation	<p>Use of examples, Planned repetition And ending</p>	<p>Solve the exercise and assignment given by the teacher in their note book</p>		<p>Use of examples, Planned repetition And ending</p>

➤ Summary

In our lesson today, we learnt that data are any numerical facts or information which can be measured or given a numerical qualification. That there are two types of data mostly used in statistics these are: discrete data and continuous data. That the number of sample in an observation may be so large that before any deductions are made about them, the raw data must be arranged in a tabular form. That apart from frequency distribution table, a clearer method of presenting data is the graphical method.

➤ Assignment

In a college Mathematics class all the students are also taking anthropology, history, or psychology and some of the students are taking two or even all three of these courses. If (i) forty students are taking anthropology, (ii) eleven students are taking history, (iii) twelve students are taking psychology, (iv) three students are taking all three courses, (v) six students are taking anthropology and history, and (vi) six students are taking psychology and anthropology. Represent the information on a pie chart.

- How many students are taking only anthropology?
- How many students are taking anthropology or history?
- How many students are taking history and anthropology, but not psychology?

APPENDIX F : POSSIBLE WRITING PROMPTS FOR MATHEMATICS IV**➤ Procedural:**

• Explain variation and how to calculate direct variation using unitary method. • Explain the distributive property to a friend who missed this class. • Explain how to construct a pie chart. Use diagrams to help your explanation. • What is an algebraic expression? When are they useful? • How do you create a table of values for a simultaneous equation?

➤ Conceptual:

• What would happen if a joint variation has only one constant of variation in a given problem? What is wrong with this? • Explain to your best friend why $4a + 5b = 9ab$ is incorrect. Draw a picture if necessary. • Why should we plot the graph of simultaneous equation for solution? Is it a useful method? • How does what we learned today apply in real life? • Jim and Shola solved quadratic equation problem of today's class in different ways. Which method would you use and why? • Sally solved today's problem in this way- _____. What is wrong with her procedure? What is a correct way to solve it? • When is 'guess and check' a useful problem-solving method? When is it not? • How are ratios and fractions related? How are they different?

➤ Affective/ Reflective:

• What was difficult to understand today? • What did you learn today? • What questions do you have about yesterday's homework? About today's lesson? • When I think of proportion, I feel.

• My favourite memory of my Mathematics career is... • My Mathematititude (attitude about Mathematics) is... because... • Should students be permitted to use calculators in class? Why or why not? • Write a poem about variations. Use the terms direct, joint, inverse partial. • What is easier to understand: ratios, fractions or proportions? Why? • How did you prepare for today's test? How do you feel you did? • What could you do to improve your marks in this course? • How would you describe your Mathematics experience in this course so far? • What can I do to help you better understand today's lesson?

APPENDIX G AND H: TEST AND EXAMINATION QUESTIONS

- A department supervisor is considering purchasing one of two comparable photocopy machines, A or B. Machine A costs #10,000, and machine B costs #10,500. This department replaces photocopy machines every three years. The repair contract for machine A costs #50 per month and covers an unlimited number of repairs. The repair contract for machine B costs #200 per repair. Based on past performance, the distribution of the number of repairs needed over any one-year period for machine B is shown below.

Number of Repairs	0	1	2	3
P r o b a b i l i t y	0 . 5	0 . 2 5	0 . 1 5	0 . 1

- ✓ You are asked to give a recommendation based on overall cost as to which machine, A or B, along with its repair contract, should be purchased. What would your recommendation be? Give a statistical justification to support your recommendation.
- ✓ Write a letter to a fictitious person that was absent from class today explaining the problem that you completed. Include the following components in your letter, the name of the type pf question given, the importance of learning this type of problems, steps taken to solve the problem and your discoveries.
- Which of the following two jobs gets better pay? (Job 1) At Tantalizers's, you will be paid #9.00 per hour and will be expected to work 20 hours per week. You are required to buy a uniform for #30. (Job 2) At Tilapia resort, you will be paid #8.50 per hour and will be expected to work 20 hours per week. There is no required special attire.
- There are 14 boys and 16 girls in Mrs Uko's class. What ratio best represent the relationship between the number of boys and the total number of students in Mrs Uko's class. (a) 7/15 (b) 7:10 (c) 7/9 (d) 14:16 (e) 8:15. Give explanation to your choice and analyze the question. 3b. If the ratio of A to B is 3:4, and the ratio of B to C is 2:3, what is the value of A when C is 5. Give analysis of why you chose the option you chose. (a) 10 (b) 2/2 (c) 5/2 (d) 5 (e) 6.
- The volume of wood in the tree (V) varies directly as the height (h) and the square of the girth (g). If the volume of the tree is 144m³ when the height is 20m and the girth is 1.5m. What is the height of a tree with a volume of 1000 and girth of 2m. Credit will be given for using more than one method to solve the problem.
- The Apostolic church Abak Usung Atai is having a fundraising for the church building. The church decided to print appeal card in form of raffle tickets. The number of tickets Eno can buy is inversely proportional to the cost of the tickets, she can afford 15 tickets that cost #500.00 each. How many tickets can Eno buy if each cost #300.00 each. Considerations will be given to those that can tackle the problem with alternative methods.
- If 14 women can cultivate 42 acres of land in 18 weeks, how many weeks will it take 21 women working at the same rate to cultivate 52 acres of land. Use methods other than the conventional method to solve the problem.
- The amount of money raised at Topfaith schools fundraising is directly proportional to the number of persons that attended the function. Last year when the same function was held, the amount of money raised for 100 attendees was #25,000. How much money will be raised this year if 1000 people attend the function. Use more than one method to solve the problem.
- The cost of attending a fair consist of a fixed gate fee of #100 and a charge for riding rides which is proportional to the number of rides ridden. If the cost of attending the fair is #240 when 7 rides are ridden, find the cost of riding 10 rides. Use more than one method to solve the problem.
- The cost of a conference party for a group of participants is partly constant and partly varies inversely as the number of participants. If the cost is #850.00 when there are 17 participants and #550.00 when there are 11 participants, find the cost when there are 20 participants. Explain every step taken to solve the problem to a friend who was absent from the class to his or her perfect understanding of the problem.
- The salary schedule for Lecturers in a university of Calabar has five steps. Here are the salaries and number of Lecturers for each step.

Step	Salary	Lecturers
1.	#25,000	42
2.	#28,000	98
3.	#35,000	138
4.	#45,000	84
5.	#55,000	38

- ✓ Construct a pie chart that represent this information (b) What is the mean salary in the university? c) A Lecturer's contribution to the school district's retirement plan is #500 plus 5% of his or her salary. What is the mean contribution? d) Write an article for your local newspaper using the information from the problem and its solutions. Note: Steps involve years of experience.
- In the drought year 1988, statements were made that over half of Sokoto state corn producers did not get back from their corn crop the money they put into seed, fertilizer, etc. The following shows the weight of the bags of corn rounded off to the nearest kilogramme recovered the producers. 62 61 65 67 54 64 55 64 67 64 68 69 75 69 65 74 75 60 71 78 59 63 66 82 61 77 61 58.

- a.) Prepare a frequency table of the weight using class size of 3 and write down the cumulative frequency table as well as cumulative percentages in your table. (b.) What is the statistical concept in this problem and why do we use class size and what determines the appropriate class size to be used. (c.) How many of the bags are below 72kg? d.) What percentage of the bags is 75kg and above?
- A man is four times as old as his son. In four years time, he will be three times as old. (a) What are their ages now. Describe how you solved this problem, step by step. (b) Give explanations to what informed you to take each step.
 - In a college Mathematics class all the students are also taking anthropology, history, or psychology and some of the students are taking two or even all three of these courses. If (i) forty students are taking anthropology, (ii) eleven students are taking history, (iii) twelve students are taking psychology, (iv) three students are taking all three courses, (v) six students are taking anthropology and history, and (vi) six students are taking psychology and anthropology.
 - ✓ How many students are taking only anthropology? b) How many students are taking anthropology or history? c) How many students are taking history and anthropology, but not psychology? (d) Represent the data on a bar chart.
 - Solve the equations $2x + 4y = 42$, $6x - 4y = 30$. (a) Which method is suitable for solving the problem. (b) Give explanation to each step you take to solve the problem.
 - The sum of two numbers is 8, their product is 15. (a) Find the numbers (b) explain how you would go about solving the problem and state what mathematical concept it represents.
 - In Danik Institute of Management, 50 students obtain the following scores in the Diploma Examination. 65 70 60 47 51 55 59 63 68 63 47 53 72 53 67 62 64 70 57 56 73 56 48 51 58 63 65 62 49 64 53 59 63 50 48 72 67 56 61 64 66 52 49 62 71 58 53 69 63 59. (a) Using these scores as example where necessary, explain what data collection is all about and why it is important. (b) Using the scores discuss tabular representation of data under the following sub-headings Frequency table and tally, class interval, class limit, class boundary, class mark and cumulative frequency table. Hint prepare the data using seven class intervals.
 - A stove costs ₦428. You make a ₦50 down payment and monthly payments of ₦42. How many months will it take to finish paying for the stove? Give explanation to your answer.
 - John solved the equation, $2^2 - x = 0$, and he got the correct answer: $x = 3/2 \pm \sqrt{15/2}$. Find the value of x . (a) use more than one method to solve. (b) Write a letter to a friend that was absent from class today explaining the problem that you completed. Include every component of the problem in your letter.
 - Solve the equation: $4(2-3)^2 - 36 = 0$. Using more than one method.
 - Suppose $2^2 - 6x = 0$ and $2^2 - 6x = 2$ are equivalent to $(-)^2 = 7$ and $(-)^2 =$, respectively. Find the value of x . Explain the steps used in solving the problem to support your answer. (b) Identify the Mathematical concept(s) that you used to solve the problem and define each one.
 - Write a paragraph (9-10 complete sentences) explaining each of the three ways to solve systems of equations. The three ways are: graphing, substitution, and elimination. Use your book and notes to help you. In your paragraph state which method you prefer and why. Then select one of the three problems below and solve it using your preferred method. Bonus: Solve the other two problems using each of the other two methods.
1. $2y = 3x + 5$, $2y - 6x = 4$ 2. $9x + 4y = -17$, $12y = -3x - 3$ 3. $5x - 7y = -21$, $14y - 5x = 22$.
- You and your friend are having trouble graphing quadratic functions of the form $y = ax^2$ and $y = ax^2 + c$. Your friend asks you to write some generalizations to help her graph these types of equations. (a) Explain the role of a . (b) Explain the maximum and minimum. (c) What is the vertex? (d) Explain the role of c .
 - Alice received a check and she decided to cash it at her local bank. For some reason, the bank teller was confused and switched the dollars and cents; that is, what was written as cents on the check he gave to her in dollars, and what was written in dollars on the check he gave to her in cents. It was not until after she bought a piece of candy for five cents that she noticed the teller's error. At that point, she actually had twice the amount of money that was written on the check. How much money was the check made out for? Explain your thinking and your answer(s). Also, explain why you believe there is only one possible answer, or why you believe there is more than one possible answer.
 - The consumption of five brands of soft drink for a week by their consumers in Lagos is as follows. Coke---4000, Sprite---3000, Guinness malt---8000, Fanta---5,500 and Pepsi---6,000. 1) Draw a bar chart representing the information. 2) Which of the brands is the highest brand consumed. 3) What is the average consumption per day to the nearest whole number. 4) What is the average soft drink consumption in a week. 5) Which of the brands of soft drink is consumed less. Give explanation in each case how you got the answer to the problem and justify the answers too.
 - What is the quadratic equation in x whose roots are $-3/2$ and 7 . Use more than one method to solve and justify the steps taken to solve the problem.

APPENDIX I: SCORING RUBRIC FOR PROBLEM-SOLVING PROCESSES

Level	Problem-Solving skills	I n c r e a s i n g Conceptual Understanding	Demonstrating Procedural Application	Developing Content connection	Demonstrating Mathematical Reasoning
4	Defines key parts of the problem or task. Develops a plan that uses a wide understanding of all aspects of the problem. Plan is creative and innovative.	Identifies and provides information about major concepts, supplies a complete or partial solution and explanation when appropriate.	Selects and executes appropriate strategies. Representations and algorithms are appropriate.	The Mathematics is accurate. All Mathematical concepts and ideas are accurately identified. Mathematical terms are used appropriately.	Compactly and accurately provides justification for major steps in process. Develops sound ideas of answer and supports the answer.
3	Defines key parts of the problem or task. Develops a plan that uses a wide understanding of key parts of the problem. Plan is creative and innovative.	Identifies and provides information about major concepts for problem solution. Key concepts are identified when appropriate for problem solution.	Selects and executes appropriate strategies. Representations and algorithms are appropriate for problem solution.	The Mathematics is accurate. Mathematical concepts and ideas are accurately identified. Mathematical terms are used appropriately for the problem.	Accurately provides justification for major steps in process but lacks depth or detail. Develops sound ideas of answer but supports the answer.
2	Defines key parts of the problem or task but misses some of the important aspects of the problem. Plan includes some understanding of problem.	Identifies and provides information about major concepts but some are missing or incomplete. Some details are given or supported in the answer.	Selects appropriate approach to solve the problem. Representations and algorithms are appropriate for the problem but some are missing or incomplete.	The Mathematics contains minor errors. Mathematical concepts and ideas are identified but not all are correct. There are some errors in development. Mathematical terms.	Provides justification for some of the steps in process but not all are correct. Develops some ideas of answer but supports the answer.

L e v e l	Problem-Solving skills	I n c r e a s i n g Conceptual Understanding	Demonstrating Procedural Application	Developing Content connection	Demonstrating Mathematical Reasoning
1	Does not identify key parts of the problem or task but responds to the problem with a limited understanding. Does not develop a plan.	Does not identify or identify major concepts and information contained in the problem or understanding.	Selects an appropriate approach to solve the problem but does not use appropriate representations and algorithms.	The Mathematics contains major errors. Mathematical concepts and ideas are identified but not all are correct. There are significant errors in development.	Provides no justification for steps in process but contains minor errors. Contains no supporting evidence and does not develop an answer.

APPENDIX J : ASW SCORING RUBRIC

Each attempted problem solving approach to the problem presented on an ASW, the pre and post tests will be scored as follows. A total score will be computed by adding up points of each attempted problem solving approach.

➤ *4 Points*

Student utilizes a correct problem solving approach, and has a correct solution.

➤ *3 Points*

Student utilizes a correct problem solving approach, but a little incompleteness or a few errors.

➤ *2 Points*

Student utilizes a correct problem solving approach but solves the problem with some incompleteness or some errors.

➤ *1 Point*

Student minimally understands the problem. It seems the students is aware of a correct problem solving approach, but a correct approach is not pursued at all.

➤ *0 Points*

Student utilizes a wrong problem solving approaches or incorrectly indentifies the problem to be solved. This student does not understand the problem.

APPENDIX K: SAMPLE OF THE ALTERNATIVE-SOLUTION WORKSHEET

P	r	o	b	l	e	m														
I	n	i	t	i	a	l	S	o	l	u	t	i	o	n	:					
A	l	t	e	r	n	a	t	i	v	e	S	o	l	u	t	i	o	n	s	: