

Special Relativity–Temporal Correction Factor

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Abstract : In the depths of special relativity, a fundamental question arises : can the direction of the thought experiment influence the time dilation factor, described by the Lorentz factor ? To answer this question, we will explore a bold thought experiment, where a light beam is emitted in a moving train, precisely in a wagon where the light will be directed towards the ceiling with an inclination relative to the train's motion. Through this study, we will unveil the secrets of spacetime and examine the fascinating implications of special relativity on our understanding of the universe, notably by introducing a new time correction factor that arises from the analysis of this thought experiment and allows to correct the limitations of the Lorentz factor, providing a more precise description of the effects of special relativity on time, which will have significant implications for clock synchronization, such as those in GPS systems.

Keywords: *Special Relativity, Thought Experiment, Lorentz Factor, Time Correction Factor, Clock Synchronization.*

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I. INTRODUCTION

The notion of relativity has evolved over the centuries, from the first reflections of Galileo (1564-1642) on the relativity of motion to the work of Isaac Newton (1643-1727) who laid the foundations of classical mechanics. The equations of electromagnetism of James Clerk Maxwell (1831-1879) then showed that the speed of light is a universal constant, which was confirmed by the experiments of Albert Michelson (1852-1931) and Edward Morley (1838-1923) in 1887. These discoveries paved the way for Albert Einstein's theory of special relativity (1879-1955), which introduced the concept of time dilation and the Lorentz factor, which describes how time is affected by the relative speed between observers. The work of Hendrik Lorentz (1853-1928) on the transformation of spatial and temporal coordinates also played a crucial role in the development of this theory. However, it is with Albert Einstein (1905) that special relativity really took off, revolutionizing our understanding of space and time. As part of this study, we will explore the limits of the Lorentz factor in the context of a thought experiment where a light beam is emitted in a moving train. By considering in particular the case where the light beam is emitted with an inclination with respect to the direction of motion of the train, we propose a new time correction factor that allows for a more precise description of the effects of special relativity in this situation. But the question arises of the impact of this new time correction factor on the synchronization of clocks, particularly in critical

applications such as GPS, where temporal precision is crucial. How will this time correction factor affect the synchronization of satellite clocks and what are the potential implications for navigation and positioning systems?

II. SPECIAL RELATIVITY OF EINSTEIN

➤ Conceptual Foundation

Let's imagine a universe where time and space are intimately linked, where the speed of light is the only constant that governs everything. This is the universe described by Albert Einstein's theory of special relativity, a scientific revolution that transformed our understanding of space and time by postulating that the speed of light is a universal constant and that the laws of physics are invariant in all inertial reference frames. This theory has opened up new perspectives on our understanding of reality, particularly through the Lorentz time dilation factor. Einstein proposed a thought experiment where a light beam is projected vertically towards the ceiling of a moving train, demonstrating the implications of special relativity on time measurement. We will explore a variant of this experiment by introducing an inclination of the light beam with respect to the direction of motion, and examine its impact on the Lorentz time dilation factor, in order to deepen our understanding of temporal phenomena and their implications for clock synchronization. A crucial aspect of modern physics.

➤ Thought Experiment of Special Relativity.

We will let ourselves be carried away by our imagination and conceive a train linked to a reference frame R' which moves horizontally in a uniform rectilinear motion at speed u relative to a reference frame R supposed to be stationary. Inside the train, a laser lamp is fixed at a point O on the bottom floor of the train and pointed towards a point A on the ceiling. Observer 1, stationary in the train, triggers an event which consists of turning on the laser lamp and starting a chronometer (T1) to measure the time it takes for the light beam to reach point A on the ceiling. It should be noted that the light beam OA makes an angle $\psi \in [0, \pi]$ with the horizontal as illustrated in Figure 1.

Outside the train, in the stationary reference frame R , Observer 2 is stationary and at the moment when the event is triggered, he starts a chronometer (T2) to measure the time it takes for the light beam to reach the ceiling of the train. The starting point of the light beam seen by observers 1 and 2 is O , and while the light beam propagates, the train travels a distance OO' , so according to Observer 2 the light beam reaches the ceiling of the train at point A' .

- For Observer 1, the light beam travels the distance OA and his chronometer (T1) measures a duration t
- For Observer 2 the light beam travels the distance OA' and his chronometer (T2) measures a duration t'
- While the beam propagates, the train travels a distance OO' in a duration t'

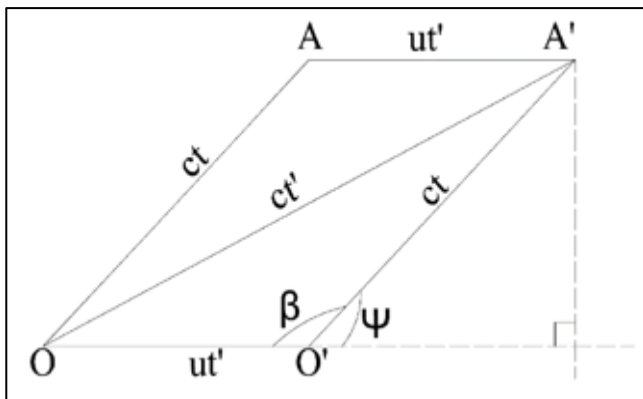


Fig 1 Description of the Trajectories and Distances Traveled by the Light Beam and the Train as Seen by Observers 1 and 2.

➤ Mathematical Formalism and Interpretation

We will focus on the triangle $O'A'O$ with vertex O' . To establish a mathematical relationship between the lengths of this triangle, we will use the law of cosines also known as Al-Kashi's theorem. Considering point O' as the vertex, we will apply this law to relate the sides OO' , OA' and $O'A'$. We can thus write:

$$(OA')^2 = (OO')^2 + (O'A')^2 - 2\|OO'\|\|O'A'\| \cos(\beta) \quad (1)$$

$$\begin{cases} \|\overline{OA'}\| = ct' \\ \|\overline{OO'}\| = ut' \\ \|\overline{O'A'}\| = ct \end{cases} \quad (2)$$

With $\beta = \pi - \psi$

(2) in (1), we can write:

$$(ct')^2 = (ut')^2 + (ct)^2 - 2(ut')(ct) \cos(\beta) \quad (3)$$

Considering t' as the unknown of this equation, we obtain a quadratic equation that we can solve to find the solutions for t' . Thus, equation (3) can be written in the following form :

$$\left(1 - \frac{u^2}{c^2}\right)(t')^2 + 2\frac{u}{c}tt' \cos(\beta) - t^2 = 0 \quad (4)$$

The discriminant of equation (4) is expressed as :

$$\begin{aligned} \Delta &= 4t^2 \frac{u^2}{c^2} \cos^2(\beta) + 4\left(1 - \frac{u^2}{c^2}\right)t^2 \\ &= 4t^2 \left(\frac{u^2}{c^2} \cos^2(\beta) + 1 - \frac{u^2}{c^2}\right) \\ &= 4t^2 \left(\frac{u^2}{c^2} (\cos^2(\beta) - 1) + 1\right) \\ \Delta &= 4t^2 \left(1 - \frac{u^2}{c^2} \sin^2(\beta)\right) \end{aligned} \quad (5)$$

We know that $\beta \in [0, \pi]$ and $u \leq c$ so Δ is positive. Therefore, the solutions are :

$$\begin{cases} t'_1 = \frac{-\frac{u}{c} \cos(\beta) + \sqrt{\left(1 - \frac{u^2}{c^2} \sin^2(\beta)\right)}}{\left(1 - \frac{u^2}{c^2}\right)} t \\ t'_2 = -\frac{\frac{u}{c} \cos(\beta) + \sqrt{\left(1 - \frac{u^2}{c^2} \sin^2(\beta)\right)}}{\left(1 - \frac{u^2}{c^2}\right)} t \end{cases} \quad (6)$$

Whit $\beta \in [0, \pi]$, it is quite obvious that t'_1 is positive and t'_2 is negative. Since time can only be positive, we will consider only the solution t'_1 .

$$t'_1 = \frac{-\frac{u}{c} \cos(\beta) + \sqrt{\left(1 - \frac{u^2}{c^2} \sin^2(\beta)\right)}}{\left(1 - \frac{u^2}{c^2}\right)} t$$

Whit $\beta = \pi - \psi$

We can thus write $t'_1 = t' = \gamma t$

$$\gamma = \frac{-\frac{u}{c} \cos(\beta) + \sqrt{\left(1 - \frac{u^2}{c^2} \sin^2(\beta)\right)}}{\left(1 - \frac{u^2}{c^2}\right)} \quad (7)$$

Where γ is the Lorentz factor.

✓ For $\psi = \frac{\pi}{2}$ $\beta = \frac{\pi}{2}$ et $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$ which is the classical

Lorentz factor established by Albert Einstein in 1905.

✓ For $\psi = 0$ $\beta = \pi$ et $\gamma = \frac{\left(1 + \frac{u}{c}\right)}{\left(1 - \frac{u^2}{c^2}\right)}$

✓ For $\psi = \pi$ $\beta = 0$ et $\gamma = \frac{\left(1 - \frac{u}{c}\right)}{\left(1 - \frac{u^2}{c^2}\right)}$

After this deep dive into the equations governing the Lorentz factor, it turns out that this factor depends not only on the speed of the moving reference frame but also on the angle of inclination of the light beam with respect to the direction of motion. The angle of inclination plays a crucial role in determining the Lorentz factor. When we set this angle to 90 degrees, we retrieve the classical Einstein equations. However, this implies that for a given reference frame in uniform rectilinear motion, we can have an infinite number of Lorentz factor values, since for each value of the angle of inclination, we obtain a different Lorentz factor. This leads to a fascinating paradox: how can we explain these contradictory results? Indeed, if we consider a train in uniform rectilinear motion with thousands of passengers on board, an observer standing on the platform would see each passenger as having a different Lorentz factor, depending on the specific orientation of the light beam used to measure their motion. This means that for this observer, time would slow down differently for each passenger on the train. How can we explain to these thousands of passengers on the same train that their time flows differently simply because they are performing thought experiments at different inclinations? This is a paradox that challenges our understanding of space and time and pushes us to rethink the foundations of special relativity. To resolve this problem, it is essential to rethink Einstein's thought experiment and add additional hypotheses that will allow us to calculate the Lorentz factor in a consistent and global manner.

III. CORRECTION OF THE LORENTZ FACTOR

➤ Hypothesis for Correcting the Lorentz Factor

In order to correct the Lorentz factor, we propose the following hypotheses to determine how time flows differently in two reference frames in motion relative to each other. Specifically, we seek to understand how time is slowed down in one reference frame relative to the other. To do this, we must define a reference reference frame with respect to which the velocities of the two reference frames to be compared will be measured.

- Hypothesis 1: Identical events :The experiments performed in the two reference frames to be compared

must be perfectly identical, unlike in section (1.2) where the experiment is performed in only one reference frame.

- Hypothesis 2: Referential velocities : The velocities to be taken into account are those of each reference frame with respect to the reference reference frame, and not the relative velocity between the two reference frames to be compared.
- Hypothesis 3: Uniqueness of the Lorentz factor :The Lorentz factor is unique for two given reference frames and does not depend on the specific parameters of the experiment performed in each reference frame, such as the angle of inclination of the light beam. It depends only on the referential velocities with respect to the reference reference frame.

➤ New Thought Experiment

We will once again let ourselves be carried away by our imagination and conceive two perfectly identical trains, called W1 and W2. Train W1 is linked to a reference frame R1 which moves in uniform rectilinear motion at a speed u relative to a reference reference frame R supposed to be stationary. Train W2 is linked to a reference frame R2 which also moves in uniform rectilinear motion, but at a speed v relative to the stationary reference reference frame R.

Considering the train W1 where an observer 1 is standing, he triggers an event which consists of turning on a laser lamp fixed at a point O on the bottom floor of the train, generating two perfectly identical and symmetrical light beams, pointed towards the ceiling of the train. The two light beams make the same angle $\psi \in [0, \pi/2]$ with the horizontal, with opposite inclinations with respect to the direction of motion of the train: one follows the direction of motion of the train and heads towards a point A1 on the ceiling, the other goes in the opposite direction of the motion of the train and heads towards a point A2 on the ceiling of the train as illustrated in Figure 2. The two light beams are emitted simultaneously and at the same moment when the event is triggered, observer 1 starts a chronometer (T) to measure the time it takes for the two light beams to reach the ceiling of the train. Outside the train, in the reference frame R, observer 3, stationary with respect to R, starts two chronometers (T'1 and T'2) at the precise moment when the event is triggered in the train W1 in order to measure the time it takes for each light beam to reach the ceiling of the train.

- OA_1 : Is the distance traveled by the light beam in the direction of the train's motion in a time t as seen by observer 1.
- OA'_1 : Is the distance traveled by the light beam in the direction of the train's motion in a time t'_1 as seen by observer 3.
- OO'_1 : Is the distance traveled by the train while the beam traveled the distance OA'_1 in the direction of the train's motion as seen by observer 3.
- OA_2 : Is the distance traveled by the light beam in the opposite direction of the train's motion in a time t as seen by observer 1.

- OA'_2 : Is the distance traveled by the light beam in the opposite direction of the train's motion in a time t'_2 as seen by observer 3.
- O'_2O : Is the distance traveled by the train while the beam traveled the distance OA'_2 in the opposite direction of the train's motion as seen by observer 3.

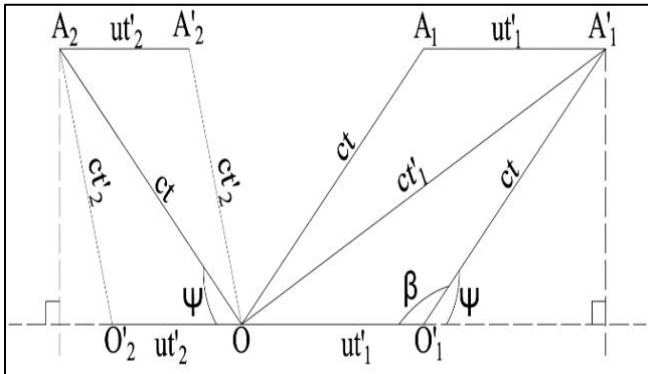


Fig 2 Description of the Trajectories and Distances Traveled by the Two Light Rays and Train W1 as Seen by Observers 1 and 3.

➤ Mathematical Formalism

We consider the triangle $O'_1A'_1O$ with vertex O'_1 and Al-Kashi's theorem is written in the following form:

$$(OA'_1)^2 = (O'_1A'_1)^2 + (OO'_1)^2 - 2\|\overrightarrow{O'_1A'_1}\| \|\overrightarrow{OO'_1}\| \cos(\pi - \psi) \quad (8)$$

$$\begin{cases} \|\overrightarrow{OA'_1}\| = ct'_1 \\ \|\overrightarrow{O'_1A'_1}\| = \|\overrightarrow{O'_1A_1}\| = ct \\ \|\overrightarrow{OO'_1}\| = \|\overrightarrow{A_1A'_1}\| = ut'_1 \end{cases} \quad (9)$$

(9) in (8), we can write:

$$(ct'_1)^2 = (ct)^2 + (ut'_1)^2 - 2(ct)(ut'_1) \cos(\pi - \psi)$$

$$(ct'_1)^2 = (ct)^2 + (ut'_1)^2 + 2(ct)(ut'_1) \cos(\psi)$$

$$\cos(\psi) = \frac{1}{2(ct)(ut'_1)} ((ct'_1)^2 - (ut'_1)^2 - (ct)^2) \quad (10)$$

We consider the triangle $OA_2O'_2$ with vertex O and Al-Kashi's theorem is written:

$$(O'_2A_2)^2 = (OA_2)^2 + (OO'_2)^2 - 2\|\overrightarrow{OA_2}\| \|\overrightarrow{OO'_2}\| \cos(\psi) \quad (11)$$

$$\begin{cases} \|\overrightarrow{O'_2A_2}\| = \|\overrightarrow{OA'_2}\| = ct'_2 \\ \|\overrightarrow{OA_2}\| = ct \\ \|\overrightarrow{OO'_2}\| = \|\overrightarrow{A'_2A_2}\| = ut'_2 \end{cases} \quad (12)$$

(12) in (11), we can Write:

$$(ct'_2)^2 = (ct)^2 + (ut'_2)^2 - 2(ct)(ut'_2) \cos(\psi)$$

$$\cos(\psi) = \frac{-1}{2(ct)(ut'_2)} ((ct'_2)^2 - (ut'_2)^2 - (ct)^2) \quad (13)$$

The equations (10) and (13) allow us to write the following system :

$$\begin{cases} \cos(\psi) = \frac{1}{2(ct)(ut'_1)} ((ct'_1)^2 - (ut'_1)^2 - (ct)^2) \\ \cos(\psi) = \frac{-1}{2(ct)(ut'_2)} ((ct'_2)^2 - (ut'_2)^2 - (ct)^2) \end{cases} \quad (14)$$

The equivalence of the two equations of the system (14) allows us to write:

$$\begin{aligned} \frac{1}{2(ct)(ut'_1)} ((ct'_1)^2 - (ut'_1)^2 - (ct)^2) &= \frac{-1}{2(ct)(ut'_2)} ((ct'_2)^2 - (ut'_2)^2 - (ct)^2) \\ \frac{1}{t'_1} ((ct'_1)^2 - (ut'_1)^2 - (ct)^2) &= \frac{-1}{t'_2} ((ct'_2)^2 - (ut'_2)^2 - (ct)^2) \\ (c^2t'_1 - u^2t'_1 - \frac{1}{(t'_1)}(ct)^2) &= -(c^2t'_2 - u^2t'_2 - \frac{1}{(t'_2)}(ct)^2) \end{aligned}$$

$$c^2(t'_1 + t'_2) - u^2(t'_1 + t'_2) = \left(\frac{1}{t'_2} + \frac{1}{t'_1}\right)(ct)^2$$

$$(c^2 - u^2)(t'_1 + t'_2) = \left(\frac{t'_1 + t'_2}{t'_2 t'_1}\right)(ct)^2$$

$$t'_2 t'_1 = \frac{c^2}{(c^2 - u^2)} t^2 \quad (15)$$

By relying on the hypothesis of perfectly identical events, the thought experiment that takes place in train W1 is reproduced in a rigorously similar manner in train W2, with the only notable distinction being that the two reference frames move at different speeds relative to the reference reference frame. Therefore, we can deduce from the above the following expression for train W2 :

$$\tau'_2 \tau'_1 = \frac{c^2}{(c^2 - v^2)} t^2 \quad (16)$$

Where

τ'_1 : is the time it takes for the light beam to reach the ceiling of the train in the direction of motion of train W2, as observed by observer 3.

τ'_2 : is the time it takes for the light beam to reach the ceiling of the train in the opposite direction of motion of train W2, as observed by observer 3.

t : is the time it takes for each of the two light beams to reach the ceiling of the train W2, as observed by observer 2 in the train W2.

v : is the speed of the reference frame R2 relative to the reference reference frame.

The equations (15) and (16) allow us to write the following system :

$$\begin{cases} t'_2 t'_1 = \frac{c^2}{(c^2 - u^2)} t^2 \\ \tau'_2 \tau'_1 = \frac{c^2}{(c^2 - v^2)} t^2 \end{cases} \quad (17)$$

$$\begin{cases} \frac{(c^2 - u^2)}{c^2} t'_2 t'_1 = t^2 \\ \frac{(c^2 - v^2)}{c^2} \tau'_2 \tau'_1 = t^2 \end{cases}$$

$$\frac{(c^2 - u^2)}{c^2} t'_2 t'_1 = \frac{(c^2 - v^2)}{c^2} \tau'_2 \tau'_1$$

$$t'_2 t'_1 = \frac{(c^2 - v^2)}{(c^2 - u^2)} \tau'_2 \tau'_1$$

$$t'_2 t'_1 = \gamma_c^2 \tau'_2 \tau'_1$$

$$\gamma_c^2 = \frac{(c^2 - v^2)}{(c^2 - u^2)}$$

$$\gamma_c = \sqrt{\frac{(c^2 - v^2)}{(c^2 - u^2)}} = \sqrt{\frac{1 - \frac{v^2}{c^2}}{1 - \frac{u^2}{c^2}}} \quad (18)$$

In the quest to resolve the problem of the dependence of the Lorentz factor on the specific parameter of the experiment, namely the angle of inclination of the light beam involved in the reference frames in motion, we have undertaken to rethink the fundamental concept of relativity. After developing the equations governing our new thought experiment, we have arrived at a new writing of the Lorentz factor. This new factor, which we will call *the time correction factor*, is given by the formula :

$$\gamma_c = \sqrt{\frac{(c^2 - v^2)}{(c^2 - u^2)}} = \sqrt{\frac{1 - \frac{v^2}{c^2}}{1 - \frac{u^2}{c^2}}}$$

Where :

u is the speed of the reference frame R1 relative to the reference reference frame R

v is the speed of the reference frame R2 relative to the reference reference frame R.

It is essential to note that when the reference frame R2 is stationary relative to the reference reference frame R, the time correction factor reduces to the classical Lorentz factor, thus confirming the consistency of our approach with the results established by Einstein in 1905.

The results obtained have direct implications for our understanding of reality, particularly here the synchronization of clocks, which reveals the fascinating complexity of time and space. It is now necessary to refer to a reference reference

frame to calculate the time correction factor. This factor allows us to synchronize clocks by taking into account the speed of each reference frame relative to the reference reference frame.

IV. CONCLUSION

In conclusion, we have just made a major advance in our understanding of special relativity. Our study has allowed us to revise the Lorentz factor by taking into account the individual speeds of the two reference frames relative to a reference reference frame. This new approach allows for a more precise description of time dilation, considering the respective speeds of the two reference frames rather than the relative speed between the two reference frames. This is a revolution in the way we approach time dilation. Indeed, we are now comparing two reference frames that host perfectly identical events, which allows us to directly compare the effects of relativity on each reference frame. The implications of this result are significant for applications that require precise clock synchronization, such as satellite navigation systems like GPS. With this new approach, we can envision a new generation of satellite navigation systems with extreme precision. The synchronization of clocks between reference frames becomes even more crucial, as the slightest error in synchronization can lead to considerable positioning errors. But a fundamental question arises: Since special relativity is limited to inertial reference frames, how will we integrate the effects of acceleration into our model? What will be the consequences for our understanding of geodesy, gravity, and spacetime, and the universe? These are just a few of the many questions that invite us to continue our research, explore new frontiers of physics, and push the boundaries of our knowledge.

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